THE NIKODYM PROPERTY AND FILTERS ON ω

TOMASZ ŻUCHOWSKI

An infinite Boolean algebra $\mathcal{A}$ is said to have the Nikodym property when every sequence of measures on $\mathcal{A}$ which is pointwise bounded is uniformly bounded. For a free filter F on ω we consider the space $N_F = \omega \cup \{p_F\}$, where ω is a discrete subspace and open neighborhoods of p_F are of the form $X \cup \{p_F\}$ for $X \in F$.

We define a property of the filter F which implies that any Boolean algebra $\mathcal{A}$ cannot have the Nikodym property when N_F is homeomorphically embedded into the Stone space $St(\mathcal{A})$ of ultrafilters on $\mathcal{A}$. We characterize this property in terms of sequences of non-negative measures on ω, and in terms of exhaustive ideals associated to density submeasures on ω. Moreover, we study the structure of the Katětov preorder on this class of filters.