Smallest subfamilies of meager ideals ensuring P-like properties

Adam Marton

Pavol Jozef Šafárik University in Košice, Faculty of Science, Institute of Mathematics, Košice, Slovakia
e-mail: adam.marton@student.upjs.sk

Abstract

Given two ideals $\mathcal{I}$, $\mathcal{J}$ on the set X we say that $\mathcal{I}$ is a P($\mathcal{J}$)-ideal (a weak P($\mathcal{J}$)-ideal) if for any countable family $\{I_n : n \in \omega\}$ of elements of $\mathcal{I}$ there is $I' \in \mathcal{I}^*$ ($I' \in \mathcal{I}^+$) such that $I_n \cap I' \in \mathcal{J}$ for all $n \in \omega$. The P($\mathcal{J}$)-property was introduced by M. Mačaj and M. Sleziak in [1] and later applied in various works.

In this talk we shall consider two cardinal invariants, namely $\mathtt{cof}^{\mathcal{J}}(\mathcal{I})$ describing smallest families ensuring P($\mathcal{J}$)-property and $\mathtt{cov}^{+}(\mathcal{I})$, an invariant closely related to the weak P(Fin)-property. We concentrate mostly on meager ideals on ω having "nice" representations on $\omega \times \omega$ which have been isolated in the literature as critical. In fact, we mostly consider ideals determined somehow by the family of functions ${}^{\omega}\omega$. We show that regarding these particular critical ideals $\mathcal{I}$, the $\mathtt{cof}^{\mathcal{J}}(\mathcal{I})$ has twofold behaviour – either collapsing all the way down to 1, or being equal to the cardinality of a smallest base of $\mathcal{I}$. However, it turns out that this is not true in general. Moreover, using $\mathtt{cov}^{+}(\mathcal{I})$ we obtain a simple way of proving strict inequalities in Katětov order

$$\mathrm{Fin} <_K \langle \mathcal{A} \rangle <_K \mathrm{Fin} \times \mathrm{Fin}$$

for ideals generated by any MAD family $\mathcal{A}$.

Acknowledgement. *This work was supported by the Slovak Research and Development Agency under the Contract no. APVV-20-0045.*

References

[1] Mačaj M. and Sleziak M., $\mathcal{I}^{\mathcal{K}}$*-convergence*, Real Anal. Exch. **36** (2010), 177–194.

[2] Marton A., P-*like properties of meager ideals and cardinal invariants*, manuscript submitted for publication.