INFINITELY LUDIC CATEGORIES

MATHEUS DUZI

We present a new categorical approach to the study of infinite games in combinatorics. With said new approach, we define categories of infinite games and are thus able to show how the following classical result from Scheepers about covering and tightness topological games can be seen as a consequence of the existence of natural transformations between the game functors.

Theorem 0.1. *If X is a $T_{3\frac{1}{2}}$ space, then*

- ALICE *has a winning strategy in* $\mathrm{G}_1(\Omega, \Omega)$ *over* $X \iff$ ALICE *has a winning strategy in* $\mathrm{G}_1(\Omega_{\bar{0}}, \Omega_{\bar{0}})$ *over* $\mathrm{C_p}(X)$ *(Theorem 13,* [1]*)*
- BOB *has a winning strategy in* $\mathrm{G}_1(\Omega, \Omega)$ *over* $X \iff$ BOB *has a winning strategy in* $\mathrm{G}_1(\Omega_{\bar{0}}, \Omega_{\bar{0}})$ *over* $\mathrm{C_p}(X)$ *(Theorem 29,* [2]*)*

We also present many aspects of the structural richness of these new game categories and characterize them in terms of other well-established categories.

This is a joint work with Paul Szeptycki and Walter Tholen. We thank FAPESP (grant number 2021/13427-9) for the financial support.

References

[1] M. Scheepers. Combinatorics of open covers (III): Games, $C_p(X)$. *Fund. Math.*, 152(3):231–254, 1997.

[2] M. Scheepers. Remarks on countable tightness. *Topology Appl.*, 161(1):407–432, 2014.

(Matheus Duzi) ICMC-USP / YORK UNIVERSITY