Sums of triples in Abelian groups, Abstract:

In 1974, Hindman proved that considering the semigroup $(\mathbb{N},+)$, for any partition $\mathbb{N} = S_0 \uplus S_1$, there exists an infinite $X \subseteq \mathbb{N}$ such that the set of its finite sums, is monochromatic, that is, contained in one of the cells.

In contrast, in 2016 Komjáth showed that, for the group $(\mathbb{R},+)$ there exists a partition $\mathbb{R} = S_0 \uplus S_1$ such that, whenever $X \subseteq \mathbb{R}$ is uncountable, not only is the set of finite sums not monochromatic, but already the set $\mathrm{FS}_2(X) := \{x+y \mid \{x,y\} \in [X]^2\}$ is not monochromatic.

These results motivate a general investigation of additive Ramsey theory in the spirit of the classical partition calculus, and which in fact for some cases are a strengthening of the classical partition calculus.

Motivated by a problem in additive Ramsey theory at $\aleph_2$, we extend Todorcevic's partitions of three-dimensional combinatorial cubes to handle additional three dimensional objects. As a corollary, we prove that the failure of the continuum hypothesis asserts that for every Abelian group G of size $\aleph_2$, there exist a coloring $G \to \mathbb{Z}$ such that, for every uncountable $X \subseteq G$ and every integer k, there exist three distinct elements x, y, z os X such that $c(x,y,z) = k$.

For further reading the article is available here: https://arxiv.org/abs/2301.01671