

Boolean valued semantics for infinitary logics

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It is well known that the completeness theorem for $L_{\omega_1\omega}$ fails with respect to Tarski semantics. Mansfield showed that it holds for $L_{\kappa\kappa}$ if one replaces Tarski semantics with boolean valued semantics. I'll talk about using forcing to improve his result in order to obtain a stronger form of boolean completeness (but only for $L_{\kappa\omega}$). Leveraging on this completeness result, one can establish the Craig interpolation property and a strong version of the omitting types theorem for $L_{\kappa\omega}$ with respect to boolean valued semantics. I'll also present a weak version of these results for the general case $L_{\kappa\lambda}$ (if one leverages instead on Mansfield's completeness theorem). All this work is based on the key notion of consistency property.