Dense metrizable subspaces in powers of Corson compacta

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A compact space X is called a *Corson compactum* if it can be homeomorphically embedded into a Σ-product of the real lines. The class of Corson compacta properly contains the class of Gul'ko compacta, which in turn properly contains the class of Eberlein compacta. Every Gul'ko compactum has a dense metrizable subspace [1], [2], while there are ZFC examples of Corson compact spaces without dense metrizable subspaces (see [2], [4]). We investigate the following question.

Problem 1. *Does there exist a Corson compactum X such that its countable power X^ω has no dense metrizable subspace?*

We show a few consistent examples of Corson compacta X such that X^ω does not contain a dense metrizable subspace. A ZFC example of such a Corson compact space is still unknown. We also prove that the existence of a ccc counterexample to above Problem 1 is equivalent to the failure of MA_{ω_1} for powerfully ccc posets. Then we give a new proof to Kunen and van Mill's theorem stating that the existence of a non-metrizable Corson compactum with a strictly positive measure is equivalent to the failure of MA_{ω_1} for measure algebras.

Our talk is based on a joint work with Santi Spadaro and Stevo Todorcevic [3].

[1] G. Gruenhage, *A note on Gul'ko compact spaces*, Proc. AMS, 100 (1987), 371–376.

[2] A. Leiderman, *Everywhere dense metrizable subspaces of Corson compacta*, Math. Notes of Acad. Science USSR, 38 (1985), 751–755.

[3] A. Leiderman, S. Spadaro, S. Todorcevic, *Dense metrizable subspaces in powers of Corson compacta*, to appear in Proc. AMS, DOI: 10.1090/proc/15885.

[4] S. Todorcevic, *Trees and linearly ordered sets*, in Handbook of Set-theoretic Topology, (eds. K. Kunen and J. E. Vaughan), North Holland, Amsterdam, 1984, 235–293.