Smital properties of Fubini products of σ-ideals

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Let $(X,+)$ be an Abelian Polish group, $\mathcal{A} \subseteq P(X)$ a σ-algebra and $\mathcal{I} \subseteq P(X)$ a nontrivial invariant σ-ideal such that

$$(\forall A \in \mathcal{I})(\exists B \in \mathcal{A} \cap \mathcal{I})(A \subseteq B).$$

If $\mathcal{A} = \mathrm{Bor}(X)$ then we say that $\mathcal{I}$ has a Borel base. Let $A + B = \{a + b : a \in A, b \in B\}$ denote the algebraic sum of sets A and B.

Definition 1. *We say that a pair $(\mathcal{A}, \mathcal{I})$ has the Smital Property if*

$$(\forall D \subseteq X)(\forall B \in \mathcal{A} \setminus \mathcal{I})(D \textit{ is dense } \to (D + B)^c \in \mathcal{I}).$$

We will consider this notion and some weaker versions of it. During the talk we will give sufficient conditions which lead to preservation of some of the Smital properties through Fubini products. In the process we will correct some errors and generalize a couple of results from [1].

We will also show that if a pair $(\mathrm{Bor}(X), \mathcal{I})$ has a very weak form of the Smital Property then (equivalently) it is maximal among nontrivial invariant σ-ideals with Borel bases. Using this fact and Fubini products we will show that there are $\mathfrak{c}$ many maximal and invariant σ-ideals on 2^ω.

These results come from [2] written together with Robert Ralowski and Szymon Żeberski.

References

[1] Bartoszewicz A., Filipczak M., Natkaniec T., On Smital properties, Topology and its Applications, vol. 158 (2011), pp. 2066-2075.

[2] Michalski M., Rałowski R., Żeberski Sz., Ideals with Smital properties, arXiv:2102.03287v2