Partitions and P-like ideals

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Given two ideals $\mathcal{I},\mathcal{J}$ on the same set M we say that $\mathcal{I}$ is a $\mathrm{P}(\mathcal{J})$-ideal if for any countable family $\{I_n : n\in\omega\}\subseteq\mathcal{I}$ there is $I\in\mathcal{I}$ such that $I_n\subseteq^{\mathcal{J}} I$ for each $n\in\omega$. The property was introduced by M. Mačaj and M. Sleziak [2], and further investigated by R. Filipów and M. Staniszewski [1] as a part of their research on various types of ideal-based convergence in topological spaces.

We consider some important ideals induced by disjoint families, namely $\mathrm{Fin}, \mathrm{Fin}\times\emptyset, \emptyset\times\mathrm{Fin}, \mathcal{S}el, \mathcal{ED}, \mathrm{Fin}\times\mathrm{Fin}$ and their isomorphic copies. In this talk, in addition to providing the basic behaviour of $\mathrm{P}(\mathcal{J})$, we discuss the role of partitions inducing $\mathcal{I}$ and $\mathcal{J}$ when $\mathcal{I}$ is a $\mathrm{P}(\mathcal{J})$-ideal. We give combinatorial characterizations of studied notion for some pairs of aforementioned ideals and discuss the importance of the particular relation $\mathcal{I}\subseteq^{\restriction}\mathcal{J}$, i.e. the condition $(\exists E\in\mathcal{I}^*)\ \mathcal{I}\restriction E\subseteq\mathcal{J}$ in characterizing $\mathrm{P}(\mathcal{J})$, e.g.

Theorem. *Let $\mathcal{A}$ be an infinite partition of $\omega\times\omega$ into infinite sets. The following statements are equivalent.*

(1) $\mathcal{S}el\subseteq^{\restriction}(\emptyset\times\mathrm{Fin})(\mathcal{A})$.

(2) *$\mathcal{S}el$ is a* $\mathrm{P}((\emptyset\times\mathrm{Fin})(\mathcal{A}))$.

(3) *There is $k\in\omega$ such that there is no m-tower of monochromatic functions*[1] *(w.r.t. $\mathcal{A}$) for every $m>k$.*

(4) $(\forall\mathcal{E}\in[{}^{\omega}\omega]^{\omega})(\exists E\in[{}^{\omega}\omega]^{<\omega})(\forall f\in\mathcal{E})(\forall A\in\mathcal{A})\ \ |(f\cap A)\setminus\bigcup E|<\omega$.

In the case of ideals $\mathcal{I},\mathcal{J}$ such that

$$\mathcal{I}\text{ is a }\mathrm{P}(\mathcal{J})\ \equiv\ \mathcal{I}\subseteq^{\restriction}\mathcal{J},$$

the $\mathrm{P}(\mathcal{J})$ property does not distinguish countable and uncountable families in some sense.

References

[1] Filipów R. and Staniszewski M., *On ideal equal convergence*, Cent. Eur. J. Math. **12** (2014), 896–910.

[2] Mačaj M. and Sleziak M., *$\mathcal{I}^{\mathcal{K}}$-convergence*, Real Anal. Exch. **36** (2010), 177–194.

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[1] Set of partial functions $g_0,\dots,g_{k-1}$ is called a ***k*-tower of monochromatic functions** (with respect to $\mathcal{A}$), if

- there are $A_0,\dots,A_{k-1}\in\mathcal{A}$ such that $g_j\subseteq A_j$ for $j<k$,
- there is $a\in[\omega]^{\omega}$ such that $\mathrm{dom}(g_j)=a$ for each $j<k$,
- $g_i\cap g_j=\emptyset$ for $i,j<k$, $i\neq j$.