CONVERGENCE OF MEASURES AND FILTERS ON ω

DAMIAN SOBOTA

For a free filter F on ω, endow the space $N_F = \omega \cup \{p_F\}$, where $p_F \notin \omega$, with the topology in which every element of ω is isolated whereas all open neighborhoods of p_F are of the form $A \cup \{p_F\}$ for $A \in F$. Spaces of the form N_F constitute the class of the simplest non-discrete Tychonoff spaces.

We provide a characterization of those filters F for which the spaces N_F, or their Čech–Stone compactifications $\beta(N_F)$, contain non-trivial convergent sequences. In the context of the celebrated Josefson–Nissenzweig theorem from Banach space theory we also completely describe those filters F for which the spaces N_F carry sequences $\langle \mu_n \colon n \in \omega \rangle$ of finitely supported signed measures satisfying the following two conditions: $\|\mu_n\| = 1$ for every $n \in \omega$, and $\mu_n(f) \to 0$ for every bounded continuous real-valued function f on N_F.

As a consequence, we obtain a description of a wide class of filters F having the following properties: (1) if X is a Tychonoff space and N_F is homeomorphic to a subspace of X, then the space $C_p^*(X)$ of bounded continuous real-valued functions on X contains a complemented copy of the space $c_0 = \{x \in \mathbb{R}^\omega \colon \ x(n) \to 0\}$ endowed with the pointwise topology, (2) if K is a compact Hausdorff space and N_F is homeomorphic to a subspace of K, then the Banach space $C(K)$ of continuous real-valued functions on K is not a Grothendieck space. The latter result generalizes the well-known fact stating that if a compact Hausdorff space K contains a non-trivial convergent sequence, then the space $C(K)$ is not Grothendieck.

This is a joint work with Witold Marciszewski.

Kurt Gödel Research Center, University of Vienna, Austria
Email address: damian.sobota@univie.ac.at