

# THE TOWER SPECTRUM

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Let  $A, B$  be infinite sets of natural numbers. Then we say that  $A$  is *almost contained* in  $B$ , written as  $A \subseteq^* B$ , iff  $A \setminus B$  is finite, i.e. all up to finitely many elements of  $A$  are also in  $B$ . A *tower* is a sequence  $\langle A_\alpha : \alpha < \delta \rangle$  of infinite sets of naturals such that  $A_\beta \subseteq^* A_\alpha$  for  $\alpha < \beta$  and that is maximal with respect to this property.

The *tower spectrum* is the set of regular cardinals  $\kappa$  so that there is a tower of length  $\kappa$ . We are going to study what sets of cardinals can consistently be realized as the tower spectrum. In particular we will focus on the  $\aleph_n$ 's.

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