

**UNCOUNTABLE EXTREMALLY DISCONNECTED GROUPS
(AND RELATED QUESTIONS)**

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At the 2018 Winter School I talked about an interplay between filters and Boolean topological groups, which is very amazing. In particular, the nonexistence of rapid (ultra)filters implies the nonexistence of countable nondiscrete extremally disconnected groups. There naturally arise the question of whether the proof can be generalized to uncountable groups and if so, whether there exist a ZFC model without uncountable rapid ultrafilters. I asked the second question two years ago, and now I know the answer, which I want to discuss. I also asked whether the well-known properties of countable ultrafilters equivalent to selectivity are also equivalent for filters (and imply being an ultrafilter); this question has also turned out to be closely related to extremally disconnected groups, and I am going to discuss it, too. Finally, I shall prove that the existence of an uncountable containing open subgroups implies that of a nondiscrete extremally disconnected group whose cardinality either is measurable or does not exceed the continuum.

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