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Selection principle $\mathrm{S}_1(\mathcal{P}, \mathcal{R})$ and the cardinal invariant $\lambda(h, \mathcal{J})$

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Let start - $\mathrm{S}_1(\mathcal{P}, \mathcal{R})$ and $\lambda(\triangle, \bigtriangledown)$

X is an $\mathrm{S}_1(\mathcal{P}, \mathcal{R})$-space[1] iff for a sequence $\langle \mathcal{U}_n : n \in \omega \rangle$ of elements of $\mathcal{P}$ we can select a set $U_n \in \mathcal{U}_n$ for each $n \in \omega$ such that $\langle U_n : n \in \omega \rangle$ is a member of $\mathcal{R}$ where $\mathcal{P}$ and $\mathcal{R}$ are some families of sets.

- Introduced by M. Scheepers (1996) in [4].

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[1] An ideal version was presented in several papers, e.g.[3, 6]...

[2] The family $\mathcal{I} \subseteq \mathcal{P}(\omega)$ is called **ideal**, if it is closed under taking subsets and finite unions and does not contain the set ω, but contains all finite subsets of ω.

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- Introduced by M. Scheepers (1996) in [4].

$$\lambda(\mathcal{I},\mathcal{J}) = \min\{|\mathcal{R}| :\ \mathcal{R}\subseteq {}^\omega\mathcal{I} \wedge (\forall\varphi\in {}^\omega\omega)(\exists\langle s(n):\ n\in\omega\rangle\in\mathcal{R})$$
$$\{n:\ \varphi(n)\in s(n)\}\in\mathcal{J}^+\}.$$

- Definition of $\lambda(\mathcal{I},\mathcal{J})$[2] by J. Šupina (2016) in [7].

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Let start - why?

- J. Šupina (2016) proved that

$$\mathrm{non}(\mathrm{S}_1(\mathcal{I}\text{-}\Gamma, \mathcal{J}\text{-}\Gamma))^3 = \lambda(\mathcal{I}, \mathcal{J}),$$

[3] The minimal cardinality of a perfectly normal space which is not an $\mathrm{S}_1(\mathcal{I}\text{-}\Gamma, \mathcal{J}\text{-}\Gamma)$.

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- Corollary (V.Š., J. Šupina 2019)

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- Nowadays, J. Šupina presented

$$\mathrm{non}(\mathrm{S}_1(\Omega^{\mathrm{ct}}, \mathcal{J}\text{-}\Gamma)) = \lambda(*, \mathcal{J}),$$

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...

[3] The minimal cardinality of a perfectly normal space which is not an $\mathrm{S}_1(\mathcal{I}\text{-}\Gamma, \mathcal{J}\text{-}\Gamma)$.

Let change - terminology

- Let $\mathcal{A} \subseteq \mathcal{P}(\omega)$.

  a sequence $s \in {}^{\omega}\mathcal{A}$ will be called an $\mathcal{A}$-**slalom**.

- Let $h \in {}^{\omega}\omega$ and $h(n) \geq 1$ for all but finitely many $n \in \omega$.

  a Fin-slalom s is an h-**slalom**[4] if $|s(n)| \leq h(n)$ for each $n \in \omega$,

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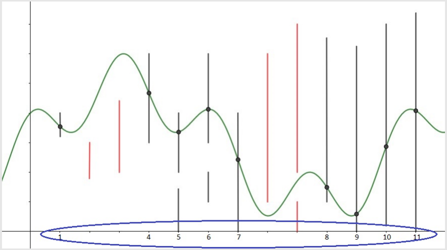

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Let change - From ideals to families

Let $\mathcal{A} \subseteq \mathcal{P}(\omega)$.

$$\lambda(\mathcal{A}, \mathcal{J}) = \min\{|\mathcal{S}| : \ \mathcal{S} \text{ consists of } \mathcal{A}\text{-slaloms, } (\forall \varphi \in {}^{\omega}\omega)(\exists s \in \mathcal{S})\ \neg(\varphi\ \mathcal{J}\text{-evades } s)\}$$

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Proposition

Let $\mathcal{A} \subseteq \mathcal{P}(\omega)$ be such family that $\bigcup \mathcal{A} = \omega$.

1. *If $\mathcal{A}$ has the finite union property and $\mathrm{Fin} \subseteq \mathcal{A}$ then*

$$\mathfrak{p} \leq \lambda(\mathcal{A}, \mathrm{Fin}) \leq \mathfrak{b}.$$

2. *If $\mathcal{A}$ does not have the finite union property then*

$$\lambda(\mathcal{A}, \mathrm{Fin}) = \min\left\{k : \ \{A_0, A_1, \dots, A_{k-1}\} \subseteq \mathcal{A} \text{ and } \bigcup_{i<k} A_i = \omega\right\}.$$

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- What about $\mathcal{A} \subseteq \mathrm{Fin}$ which has the finite union property?

Let change - from ideals to h-slaloms

- Let a function $h \in {}^{\omega}\omega$ not be a $\mathcal{J}$-**equal to zero**, i.e. $\{n : \ h(n) \neq 0\} \notin \mathcal{J}$.[5]

[5] Recall that a function φ $\mathcal{J}$-evades slalom s iff $\{n : \ \varphi(n) \in s(n)\} \notin \mathcal{J}\}$.
[6] Compare with [1] or [5].

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- By T. Bartoszyński [1] (1984)

$$\mathrm{non}(\mathcal{M}) = \min\{|\mathcal{F}| : \ \mathcal{F} \subseteq {}^{\omega}\omega, \ (\forall \varphi \in {}^{\omega}\omega)(\exists f \in \mathcal{F}) \, |\{i : \ \varphi(i) = f(i)\}| = \aleph_0\}.$$

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- Consequently,

$\lambda(h, \mathrm{Fin}) = \mathrm{non}(\mathcal{M})$ for any admissible $h \in {}^{\omega}\omega$.[6]

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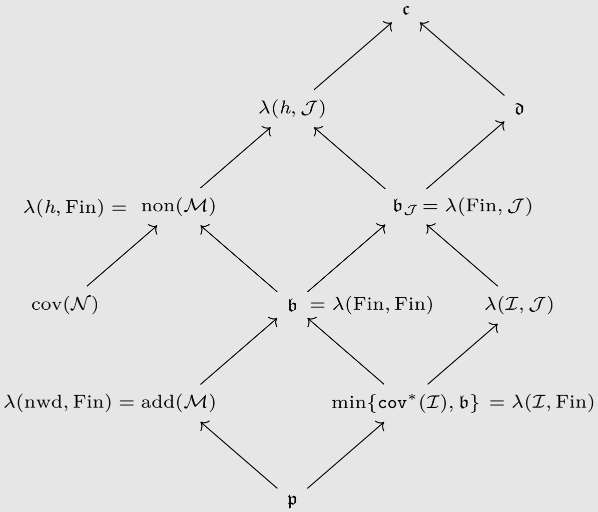

Diagram. Cardinal invariants of the continuum and the $\lambda(\mathcal{S}, \mathcal{J})$.

Let make a cover...

Recall

- a sequence $\langle U_n : n \in \omega \rangle$ of open sets of X is a $\mathcal{I}$-γ-**cover** of X iff the set $\{n \in \omega : x \notin U_n\} \in \mathcal{I}$ for each $x \in X$,
- $\mathcal{I}$-Γ denotes the family of all $\mathcal{I}$-γ-covers of X.[7]

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Let $c \in \omega$ be a constant.

- A sequence $\langle U_n : n \in \omega \rangle$ is called a γ_c-**cover** iff $|\{n \in \omega : x \notin U_n\}| \leq c$ for each $x \in X$.
  - For instance, let $\{x_n : n \in \omega\}$ be a set of pairwise disjoint point of X and define $U_n = X \setminus \{x_n\}$. Then $\langle U_n : n \in \omega \rangle$ is a γ_1-cover.
- Γ^c denotes the family of all γ_c-covers of X.

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$\mathrm{S}_1(\Gamma_h, \mathcal{J}\text{-}\Gamma)$

Let $h \in {}^\omega\omega$

- Γ_h denotes the family of all sequences of $\gamma_{h(n)}$-covers for a function i.e.,

$$\langle\langle U_{n,m} : \ m \in \omega\rangle : \ n \in \omega\rangle \in \Gamma_h$$
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$$\langle U_{n,m} : \ m \in \omega\rangle \text{ is a } \gamma_{h(n)}\text{-cover for each } n \in \omega.$$

Lemma

- *Let X be a topological space. If $|X| < \lambda(h, \mathcal{J})$ then X is an $\mathrm{S}_1(\Gamma_h, \mathcal{J}\text{-}\Gamma)$-space.*
- *Let D be a discrete topological space and $h \in {}^\omega\omega$ being no $\mathcal{J}$-equal to zero. Then $|D| < \lambda(h, \mathcal{J})$ if and only if D is an $\mathrm{S}_1(\Gamma_h, \mathcal{J}\text{-}\Gamma)$-space.*

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Corollary

$\mathrm{non}(\mathrm{S}_1(\Gamma_h, \mathcal{J}\text{-}\Gamma)) = \lambda(h, \mathcal{J})$. *In particular,* $\mathrm{non}(\mathrm{S}_1(\Gamma_h, \mathrm{Fin})) = \mathrm{non}(\mathcal{M})$.

$\mathrm{S}_1(\Gamma_h, \Gamma)$

$$
\begin{array}{ccc}
\underset{\mathrm{non}(\mathcal{M})}{\mathrm{S}_1(\Gamma_h, \Gamma)} & \longrightarrow & \underset{\lambda(h, \mathcal{J})}{\mathrm{S}_1(\Gamma_h, \mathcal{J}\text{-}\Gamma)} \\
\uparrow & & \uparrow \\
\underset{\mathfrak{b}}{\mathrm{S}_1(\Gamma, \Gamma)} & \longrightarrow & \underset{\mathfrak{b}_{\mathcal{J}}}{\mathrm{S}_1(\Gamma, \mathcal{J}\text{-}\Gamma)}
\end{array}
$$

Diagram. Relations with respect to well-known $\mathrm{S}_1(\Gamma, \Gamma)$-space.

$\mathrm{S}_1(\Gamma_h, \Gamma)$

$$
\begin{array}{ccc}
\begin{array}{c}\mathrm{S}_1(\Gamma_h, \Gamma) \\ \mathrm{non}(\mathcal{M})\end{array} & \longrightarrow & \begin{array}{c}\mathrm{S}_1(\Gamma_h, \mathcal{J}\text{-}\Gamma) \\ \lambda(h, \mathcal{J})\end{array} \\
\uparrow & & \uparrow \\
\begin{array}{c}\mathrm{S}_1(\Gamma, \Gamma) \\ \mathfrak{b}\end{array} & \longrightarrow & \begin{array}{c}\mathrm{S}_1(\Gamma, \mathcal{J}\text{-}\Gamma) \\ \mathfrak{b}_{\mathcal{J}}\end{array}
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Proposition

The Baire space is not an $\mathrm{S}_1(\Gamma_h, \Gamma)$*-space.*

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The Baire space is not an $\mathrm{S}_1(\Gamma_h, \Gamma)$*-space.*

- even more the Baire space is not an $\mathrm{S}_1(\Gamma_{\mathbf{1}}, \Gamma)$-space,

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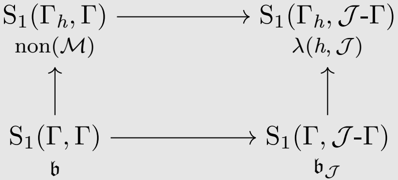

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Proposition

The Baire space is not an $\mathrm{S}_1(\Gamma_h, \Gamma)$-space.

- even more the Baire space is not an $\mathrm{S}_1(\Gamma_{\mathbf{1}}, \Gamma)$-space,
- $\lambda(h, \mathcal{J}) = \mathrm{non}(\mathcal{M})$ where $h \in {}^{\omega}\omega$ is not $\mathcal{J}$-equal to zero and $\mathcal{J}$ is an ideal that has the Baire property.

$\mathrm{S}_1(\Gamma^c, \Gamma^c)$ as a weird phenomenon...

- A set $\{X_n \subseteq X : \ n \in \omega\}$ is k-**wise disjoint** iff any k-tuple of sets has the empty intersection.[8]

[8] Let stress that notions k-wise disjoint sets and sequences will be interchangeable, i.e., a sequence $\langle X_n : \ n \in \omega \rangle$ is k-wise disjoint if and only if the corresponding set $\{X_n \subseteq X : \ n \in \omega\}$ is k-wise disjoint.

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- A set $\{X_n \subseteq X : n \in \omega\}$ is k-**wise disjoint** iff any k-tuple of sets has the empty intersection.[8]
- **characterization of γ_c-covers:**
  A sequence $\langle U_n : n \in \omega \rangle$ of open sets is a γ_c-cover if and only if $\{X \setminus U_n : n \in \omega\}$ is a $(c+1)$-wise disjoint set of closed sets in X.

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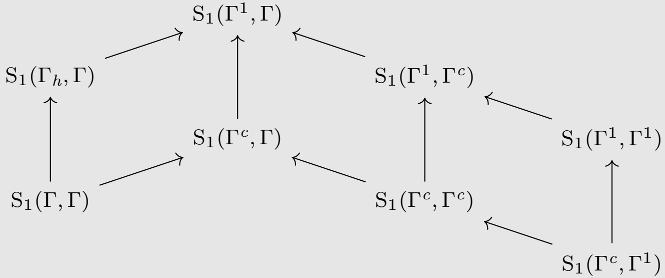

Diagram. Relations with respect to well-known $S_1(\Gamma, \Gamma)$-space.

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Proposition

Let c be a constant.

- *No infinite discrete space is an $\mathrm{S}_1(\Gamma^1, \Gamma^c)$-space.*
- *Let X be a space with infinitely many accumulation points. Then X is not an $\mathrm{S}_1(\Gamma^1, \Gamma^1)$-space.*

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Let c be a constant.

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- *Let X be a space with infinitely many accumulation points. Then X is not an $S_1(\Gamma^1, \Gamma^1)$-space.*
- *Each space with finite many but at least one accumulation points is an $S_1(\Gamma^c, \Gamma^1)$-space.*

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Let c be a constant.

- *No infinite discrete space is an $S_1(\Gamma^1, \Gamma^c)$-space.*
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- *Each space with finite many but at least one accumulation points is an $S_1(\Gamma^c, \Gamma^1)$-space.*
- *There is an $S_1(\Gamma^1, \Gamma^2)$-space which is not an $S_1(\Gamma^1, \Gamma^1)$-space.*

Thank you for your attention

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