A topological game on the space of ultrafilters

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Consider $\beta\omega$, the space of ultrafilters over the natural numbers. For a set $T \subset \omega^*$ we define the game for two players, *Alice* and *Bob*, who take turns alternately. The game starts with *Alice* choosing a natural number $a_0 \in \omega$ and *Bob* responding with another number $b_0 \neq a_0$. In the $n$-th inning, *Alice* chooses a number $a_n \in \omega$ that has not been chosen previously, that is, $a_n \notin \{a_0, b_0, \ldots, a_{k-1}, b_{k-1}\}$, similarly *Bob* responds with $b_n \in \omega$ such that $b_n \notin \{a_0, b_0, \ldots, a_{k-1}, b_{k-1}, a_k\}$. *Alice* wins the game if $\{a_k \mid k \in \omega\} \cap T \neq \emptyset$, that means the closure of the set of her choices during the game seen as a subset of $\beta\omega$ intercepts the initial target $T$, and *Bob* wins otherwise.

This presentation will cover some questions and results regarding the previous game as well as some of its relations with other infinite games.