

# A topological game on the space of ultrafilters

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Consider  $\beta\omega$ , the space of ultrafilters over the natural numbers. For a set  $T \subset \omega^*$  we define the game for two players, *Alice* and *Bob*, who take turns alternately. The game starts with *Alice* choosing a natural number  $a_0 \in \omega$  and *Bob* responding with another number  $b_0 \neq a_0$ . In the  $n$ -th inning, *Alice* chooses a number  $a_n \in \omega$  that has not been chosen previously, that is,  $a_n \notin \{a_0, b_0, \dots, a_{n-1}, b_{n-1}\}$ , similarly *Bob* responds with  $b_n \in \omega$  such that  $b_n \notin \{a_0, b_0, \dots, a_{n-1}, b_{n-1}, a_n\}$ . *Alice* wins the game if  $\overline{\{a_k \mid k \in \omega\}} \cap T \neq \emptyset$ , that means the closure of the set of her choices during the game seen as a subset of  $\beta\omega$  intercepts the initial target  $T$ , and *Bob* wins otherwise.

This presentation will cover some questions and results regarding the previous game as well as some of its relations with other infinite games.