Fréchet-like properties and ad families

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Fréchet, α_i and strong Fréchet properties

A point $x \in X$ is a Fréchet point if whenever $x \in \overline{A}$ there is a sequence $\{x_n : n \in \omega\} \subseteq A$ such that $x_n \to x$.

Definition (Arhangel'skii, 79)

A point $x \in X$ is an α_i-point $(i = 1, 2, 3, 4)$ if given a family $\{S_n : n \in \omega\}$ of sequences converging to x, there is a sequence $S \to x$ (we identify a convergent sequence with its range) such that:

(α_1) $S \setminus S_n$ is finite for all $n \in \omega$,

(α_2) $S \cap S_n \neq \emptyset$ for all $n \in \omega$,

(α_3) $|S \cap S_n| = \omega$ for infinitely many $n \in \omega$,

(α_4) $S \cap S_n \neq \emptyset$ for infinitely many $n \in \omega$.

Then a space X is Fréchet (resp. α_i) if every point $x \in X$ is Fréchet (resp. α_i).

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Fréchet, α_i and strong Fréchet properties

Definition (Arhangel'skii, 79)

A space X is absolutely Fréchet if in some Hausdorff compactification bX of X, every point $x \in X$ is a Fréchet point.

Given a family $\mathcal{A} \subseteq \mathcal{P}(X)$ we will say that $x \in \overline{\mathcal{A}}$ (x clusters at $\mathcal{A}$) if $x \in \overline{A}$ for every $A \in \mathcal{A}$. A filter base $\mathcal{G}$ converges to a point $x \in X$ if for every neighborhood V of x, there is a $G \in \mathcal{G}$ such that $G \subseteq V$. We then write $\mathcal{G} \to x$.

Definition (E. Michael, 72)

X is bisequential at $x \in X$ if for every ultrafilter $\mathcal{U}$ in X such that $x \in \overline{\mathcal{U}}$ there is a sequence $\mathcal{G} = \{G_n : n \in \omega\} \subseteq \mathcal{U}$ such that $\mathcal{G} \to x$. A space X is bisequential if it is bisequential at every point.

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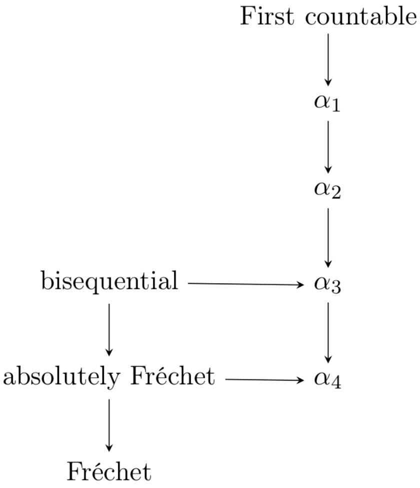
Relationship between strong Fréchet and α_1 properties



Impact of these properties in the product of Fréchet spaces

- The product of a countably compact space with an α_3-FU space is Fréchet.
- $X \times [0,1]$ is Fréchet iff X is α_4-FU.
- If X is absolutely Fréchet and Y is first countable then $X \times Y$ is absolutely Fréchet.
- The product of an absolutely Fréchet and a bisequential space is absolutely Fréchet.

Some results

- (Arhangel'skii, 79) There is a Fréchet space which is not α_4.
- (Arhangel'skii, 79) There is a non-bisequential space X such that it is α_4-FU.
- (Simon, 80) There is an α_4 space which is not α_3.
- (Nyikos, 89) There is an α_3 space which is not α_2.
- (Nyikos, 90's) There is an α_2 space that is not first countable.
- (Dow, 90's) It is consistent that all countable α_2 spaces are α_1.
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AD spaces

A family $\mathcal{A} \subseteq [\omega]^\omega$ is an almost disjoint (ad) family if $|A \cap B| < \omega$ for every $A, B \in \mathcal{A}$. $\mathcal{A}$ is maximal almost disjoint (mad) if it is ad and maximal with respect to this property.

Given an ad family $\mathcal{A}$, the ad space generated by $\mathcal{A}$ is the subspace $\omega \cup \{\infty\}$ of the one-point compactification of $\Psi(\mathcal{A})$.

We will say that an ad family $\mathcal{A}$ satisfies a topological property P if its ad space does.

Definition

An ad family $\mathcal{A}$ is hereditarily α_3 if $\mathcal{B}$ is α_3 for every $\mathcal{B} \subseteq \mathcal{A}$.

Question (Gruenhage, 06)

For an ad family $\mathcal{A}$ is it equivalent being α_3-FU (hereditarily α_3-FU) with its bisequentiality?

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- (Folklore) There are bisequential ad families (actually, every $\mathbb{R}$-embedable is bisequential).
- (Nyikos, 09) Under $\mathfrak{b} = \mathfrak{c}$, there is a Fréchet ad family which is not α_3. The example consists of graph of functions on $\omega \times \omega$, so...

Question (Nyikos, 09)

Is there a non-bisequential ad family consisting of functions such that it is α_3-FU?

Theorem (C.-Hrušák)

$\mathrm{non}(\mathcal{M}) = \mathfrak{c}$. There exists an α_3-FU (even α_2-FU) ad family (consisting of functions in $\omega \times \omega$) which is not hereditarily α_3.

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Theorem (C.-Hrušák)

$\mathfrak{b} = \mathfrak{c}$. There exists an hereditarily α_3-FU almost disjoint family (consisting of partial functions) which is not bisequential.

Question

Can the above family consist of total functions?

Since $\mathfrak{b} \leq \mathrm{non}(\mathcal{M})$, it follows that under $\mathfrak{b} = \mathfrak{c}$ the tree concepts are different.

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Other constructions

Theorem (C.-Hrušák)

($\mathfrak{s} \leq \mathfrak{b}$) There is an α_3-FU not hereditarily α_3 ad family.

Corollary

($\mathfrak{c} \leq \aleph_2$) There is an α_3-FU non-bisequential ad family.

Theorem (C.-Hrušák)

- $\diamondsuit(\mathfrak{b}) \Rightarrow$ There is an α_2-FU not her. α_3 ad family of size ω_1.
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On some questions of Arhangel'skii

- (79) Is there an absolutely Fréchet space which is not bisequential?
- (79) Is there a (countable) α_1-FU space which is not bisequential?

A consistent example for the second question was given by Malyhin under the assumption $2^{\aleph_0} < 2^{\aleph_1}$.

Theorem (C.-Hrušák)

CH. There is a countable α_1 and absolutely Fréchet space which is not bisequential.

Theorem

There is (in ZFC) an absolutely Fréchet space which is not bisequential.

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Some questions (if time allows)

- Is there (in ZFC) an α_3-FU non-bisequential ad family?

Definition

Let $\mathcal{A}$ be an ad family:

- $\mathcal{A}$ is completely separable if for every $X \in \mathcal{I}(\mathcal{A})^+$ there exists $A \in \mathcal{A}$ such that $A \subseteq X$.
- $\mathcal{A}$ is almost completely separable if for every $X \subseteq \omega$ such that $X \cap A$ is infinite for infinitely many $A \in \mathcal{A}$, there exists $B \in \mathcal{A}$ such that $B \subseteq X$
- $\mathcal{A}$ is weakly tight if for every family $\{X_n : n \in \omega\} \subseteq \mathcal{I}(\mathcal{A})^+$, there is $A \in \mathcal{A}$ such that $A \cap X_n$ is infinite for infinitely many $n \in \omega$.

- Is there (in ZFC) an almost weakly tight ad family?
- Does it follow from the existence of the above family, the existence of an α_3-FU non-bisequential ad family?

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- $\mathcal{A}$ is completely separable if for every $X \in \mathcal{I}(\mathcal{A})^+$ there exists $A \in \mathcal{A}$ such that $A \subseteq X$.
- $\mathcal{A}$ is almost completely separable if for every $X \subseteq \omega$ such that $X \cap A$ is infinite for infinitely many $A \in \mathcal{A}$, there exists $B \in \mathcal{A}$ such that $B \subseteq X$
- $\mathcal{A}$ is weakly tight if for every family $\{X_n : n \in \omega\} \subseteq \mathcal{I}(\mathcal{A})^+$, there is $A \in \mathcal{A}$ such that $A \cap X_n$ is infinite for infinitely many $n \in \omega$.

- Is there (in ZFC) an almost weakly tight ad family?
- Does it follow from the existence of the above family, the existence of an α_3-FU non-bisequential ad family?

Some questions (if time allows)

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Thank you for your attention!