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The Uniform Subsets of the Euclidean Plane

M.Beriashvili

The problem of Luzin and the theorem of Sierpinski have found interesting connections with the measure extension problem. The study of the measurability properties of uniform sets is an interesting topic for our research. In measure theory it is well known the standard concept of measurability of sets and functions with respect to a fixed measure μ on a base (ground) set E. Now we introduce the concept of measurability of sets and functions not with respect to a fixed measure μ, but with respect to certain classes of measures, which are defined on different σ-algebras of subsets of base space E. (see [1], [2]).

Let E be a set and let M be a class of measures on E (in general, we do not assume that measures belonging to M are defined on the one and same σ-algebra of subset of E).

Definition.

- We say that a function $f : E \to \mathbf{R}$ is absolutely (or universally) measurable with respect to M if f is measurable with respect to all measures from M.
- We say that a function $f : E \to \mathbf{R}$ is relatively measurable with respect to M if there exists at least one measure μ from M such that f is μ-measurable.
- We say that a function $f : E \to \mathbf{R}$ is absolutely nonmeasurable with respect to M if there exists no measure μ from M such that f is μ-measurable.

In particular, the graph of a function $\phi : \mathbf{R} \to \mathbf{R}$, which yields a positive solution of Luzin's problem, is an absolutely nonmeasurable subset of $E = \mathbf{R^2}$ with respect to the class of all nonzero σ-finite measures on $\mathbf{R^2}$ that are invariant under the group of all isometries of $\mathbf{R^2}$.

Theorem. There exists a uniform subset of $\mathbf{R^2}$ which is a Hamel basis of $\mathbf{R^2}$.

References

1. A. Kharazishvili, *One property of Hamel bases*, Bull. Acad. Sci. GSSR, 95, 2 (1979),
2. A. Kharazishvili *Questions in the theory of sets and in measure theory* , TSU, Tbilisi, 1978

Supported by SRNSFG Grant Numb: MG TG-19-1447