Kunen's inconsistency theorem can be generalized as follows:

**Theorem 1.** Suppose \( j : M \rightarrow N \) is a nontrivial elementary embedding between models of ZFC with the same ordinals. Then at least one of the following holds:

1. There is \( \alpha \in M \) such that \( \langle \alpha, j(\alpha), j^2(\alpha), j^3(\alpha), \ldots \rangle \notin M \).
2. There is \( \alpha \in M \) such that \( j[\alpha] \notin N \).
3. Some ordinal is regular in \( M \) and singular in \( N \).

We investigate embeddings that come as close as possible to the boundaries imposed by the above theorem. One example shows that an inner model which is arbitrarily close to \( V \) may be self-embeddable.

**Theorem 2.** Suppose \( \kappa \leq \lambda \) are regular. \( \kappa \) is \( \lambda \)-supercompact if and only if there is a \( \lambda \)-closed transitive class \( M \) and a nontrivial elementary \( j : M \rightarrow M \) with critical point \( \kappa \) and \( \lambda < j(\kappa) \).

We say \( j : M \rightarrow N \) is **amenable** when alternative (2) above fails, which implies \( M \subseteq N \). We give examples of such embeddings with various fixed-point properties, and also investigate the structure of the concrete categories of systems of models with the same ordinals and amenable maps between them. Let \( \text{AmOut}(M) \) be the category of models \( N \) with the same ordinals as \( M \) such that there is an amenable \( j : M \rightarrow N \), with the arrows being all amenable embeddings between these objects.

Partial and linear orders are naturally represented as categories. A pseudotree is a partial order that is linearly ordered below any given element. We define a canonical countable pseudotree that contains every other countable pseudotree as a substructure. We show:

**Theorem 3.** Suppose there is a countable transitive model of ZFC plus a measurable cardinal. Then there are many countable transitive \( M \models \text{ZFC} \) such that:

1. For every linear order \( L \), \( \text{AmOut}(M) \) has a subcategory isomorphic to \( L \) iff \( L \) is countable.
2. For every countable partial order \( P \), there is a subcategory of \( \text{AmOut}(M) \) isomorphic to \( P \).
3. There is an incompatibility-preserving injective functor from the "universal countable pseudotree" into \( \text{AmOut}(M) \).

There are many natural questions about these categories which we do not know how to answer at present.

This is joint work with Sy Friedman.