Hereditary (bi)coreflective subcategories in certain categories of semitopological groups

Veronika Pitrová

Jan Evangelista Purkyně University in Ústí nad Labem

Winter School in Abstract Analysis 2019
Semitopological groups

- $(G, \circ)$ – group with a topology
Semitopological groups

- $(G, \circ)$ – group with a topology
  - $\circ : G \times G \to G$ is separately continuous
Semitopological groups

- \((G, \circ)\) – group with a topology
  \(\circ : G \times G \to G\) is separately continuous

\(\forall g \in G : (g, x) \mapsto g \circ x\)
\((x, g) \mapsto x \circ g\) are continuous
Semitopological groups

- $(G, \circ)$ – group with a topology
  - $\circ : G \times G \to G$ is separately continuous
  - $\forall g \in G : (g, x) \mapsto g \circ x$
  - $(x, g) \mapsto x \circ g$ are continuous

- **STopGr** – the category of all semitopological groups and continuous homomorphisms
Semitopological groups

- \((G, \circ)\) – group with a topology
  - \(\circ : G \times G \to G\) is separately continuous
  - \(\forall g \in G : (g, x) \mapsto g \circ x\)
  - \((x, g) \mapsto x \circ g\) are continuous

- \textbf{STopGr} – the category of all semitopological groups and continuous homomorphisms

- all maps are continuous homomorphisms
Semitopological groups

- $(G, \circ)$ – group with a topology
  - $\circ : G \times G \to G$ is separately continuous
  - $\forall g \in G : (g, x) \mapsto g \circ x$
  - $(x, g) \mapsto x \circ g$ are continuous

- $\text{STopGr}$ – the category of all semitopological groups and continuous homomorphisms

- all maps are continuous homomorphisms

- every subcategory of $\text{STopGr}$ is full – subcategories are determined by their classes of objects
Semitopological groups

- $(G, \circ)$ – group with a topology
  $\circ : G \times G \to G$ is separately continuous
  $\forall g \in G : (g, x) \mapsto g \circ x$
  $(x, g) \mapsto x \circ g$ are continuous

- $\mathbf{STopGr}$ – the category of all semitopological groups and continuous homomorphisms
  - all maps are continuous homomorphisms
  - every subcategory of $\mathbf{STopGr}$ is full – subcategories are determined by their classes of objects
  - every subcategory of $\mathbf{STopGr}$ is isomorphism-closed
Epireflective subcategories

- $A \subseteq \text{STopGr}$ is reflective in $\text{STopGr}$:
Epireflective subcategories

- \( A \subseteq \text{STopGr} \) is reflective in \( \text{STopGr} \):
  \[
  \forall G \in \text{STopGr} \; \exists H \in A, \; r : G \to H
  \]
  such that the following diagram commutes:

  \[
  \begin{array}{ccc}
  G & \xrightarrow{f} & H \\
  \downarrow{r} & & \downarrow{! \bar{f}} \\
  H' & \xrightarrow{f} & H'
  \end{array}
  \]

- \( \text{epireflective: every reflection is an epimorphism} \)
- \( \text{extremal epireflective: every reflection is an extremal epimorphism (quotient map)} \)
Epireflective subcategories

- $A \subseteq \text{STopGr}$ is reflective in $\text{STopGr}$:

  $\forall G \in \text{STopGr} \ \exists H \in A, \ r : G \to H :$

  $\forall H' \in A \ \forall f : G \to H' \ \exists! \bar{f} : H \to H'$, such that the following diagram commutes:

  \[
  \begin{array}{ccc}
  G & \xrightarrow{r} & H \\
  \downarrow{f} & & \downarrow{\bar{f}} \\
  H' & & 
  \end{array}
  \]
Epireflective subcategories

- $A \subseteq \text{STopGr}$ is reflective in $\text{STopGr}$:
  \[ \forall G \in \text{STopGr} \ \exists H \in A, \ r : G \to H : \]
  \[ \forall H' \in A \ \forall f : G \to H' \ \exists! \bar{f} : H \to H', \text{ such that the following diagram commutes:} \]

\[
\begin{array}{ccc}
  G & \xrightarrow{r} & H \\
  \downarrow{f} & & \downarrow{\bar{f}} \\
  H' & & \\
\end{array}
\]

- epireflective: every reflection is an epimorphism
A \subseteq \text{STopGr} \text{ is reflective in } \text{STopGr}:

\forall G \in \text{STopGr} \ \exists H \in A, \ r: G \to H :
\forall H' \in A \ \forall f: G \to H' \ \exists! \bar{f}: H \to H', \text{ such that the following diagram commutes:}

\begin{align*}
G & \xrightarrow{r} H \\
\downarrow f & \quad \quad \downarrow \bar{f} \\
H' & \end{align*}

- epireflective: every reflection is an epimorphism
- extremal epireflective: every reflection is an extremal epimorphism (quotient map)
Epireflective subcategories of $\text{STopGr}$

- epireflective $\iff$ closed under the formation of products and subgroups
Epireflective subcategories of $\text{STopGr}$

- epireflective $\Leftrightarrow$ closed under the formation of products and subgroups
  - quasitopological groups ($\text{QTopGr}$)
  - paratopological groups ($\text{PTopGr}$)
  - topological groups ($\text{TopGr}$)
  - extremal epireflective $\Leftrightarrow$ closed under the formation of products, subgroups and semitopological groups with finer topologies
    - abelian semitopological groups ($\text{STopAb}$)
    - torsion-free semitopological groups
    - Hausdorff semitopological groups
Epireflective subcategories of $\text{STopGr}$

- epireflective $\iff$ closed under the formation of products and subgroups
  - quasitopological groups ($\text{QTopGr}$)
  - paratopological groups ($\text{PTopGr}$)
  - topological groups ($\text{TopGr}$)
- extremal epireflective $\iff$ closed under the formation of products, subgroups and semitopological groups with finer topologies
Epireflective subcategories of $\textit{STopGr}$

- epireflective $\iff$ closed under the formation of products and subgroups
  - quasitopological groups ($\textit{QTopGr}$)
  - paratopological groups ($\textit{PTopGr}$)
  - topological groups ($\textit{TopGr}$)
- extremal epireflective $\iff$ closed under the formation of products, subgroups and semitopological groups with finer topologies
  - abelian semitopological groups ($\textit{STopAb}$)
  - torsion-free semitopological groups
  - Hausdorff semitopological groups
Coreflective subcategories

\[ B \subseteq A \] is coreflective in \( A \):
Coreflective subcategories

- \( \mathcal{B} \subseteq \mathcal{A} \) is coreflective in \( \mathcal{A} \):
  \[
  \forall G \in \mathcal{A} \ \exists H \in \mathcal{B}, \ c : H \to G
  \]

Monocoreflective: every coreflection is a monomorphism

Bicoreflective: every coreflection is a bimorphism (monomorphism and epimorphism)

Veronika Pitrová
Coreflective subcategories

- \( B \subseteq A \) is coreflective in \( A \):

  \[
  \forall G \in A \ \exists H \in B, \ c : H \to G :
  \forall H' \in B \ \forall f : H' \to G \ \exists! \bar{f} : H' \to H, \text{ such that the following diagram commutes:}
  \]

  \[
  \begin{array}{ccc}
  H & \xrightarrow{c} & G \\
  \uparrow \bar{f} & & \downarrow f \\
  H' & \xrightarrow{f} & H \\
  \end{array}
  \]

  monocoreflective: every coreflection is a monomorphism
  bicoreflective: every coreflection is a bimorphism (monomorphism and epimorphism)

Veronika Pitrová
Coreflective subcategories

- \( B \subseteq A \) is coreflective in \( A \):

\[
\forall G \in A \; \exists H \in B, \; c : H \to G : \forall H' \in B \; \forall f : H' \to G \; \exists! \; \tilde{f} : H' \to H, \text{ such that the following diagram commutes:}
\]

\[
\begin{array}{ccc}
H & \xrightarrow{c} & G \\
\uparrow \tilde{f} & & \downarrow f \\
H' & & \\
\end{array}
\]

- monocoreflective: every coreflection is a monomorphism
Coreflective subcategories

- \( B \subseteq A \) is coreflective in \( A \):
  \[
  \forall G \in A \ \exists H \in B, \ c : H \to G :
  \forall H' \in B \ \forall f : H' \to G \ \exists! \ \bar{f} : H' \to H, \text{ such that the following diagram commutes:}
  \]

\[
\begin{array}{ccc}
H & \xrightarrow{c} & G \\
\uparrow \bar{f} & & \downarrow f \\
H' & \xrightarrow{f} & \\
\end{array}
\]

- monocoreflective: every coreflection is a monomorphism
- bicoreflective: every coreflection is a bimorphism (monomorphism and epimorphism)
Coreflective subcategories in \( \mathbf{A} \)

- monocoreflective \( \iff \) closed under the formation of coproducts and extremal quotients
Coreflective subcategories in $\mathbf{A}$

- monocoreflective $\iff$ closed under the formation of coproducts and extremal quotients
  - extremal quotient: $G_1 \overset{q}{\rightarrow} G_2$
- monocoreflective $\Leftrightarrow$ closed under the formation of coproducts and extremal quotients
  - extremal quotient: $G_1 \xrightarrow{q} G_2 \xrightarrow{r} G_3$
Coreflective subcategories in $\mathbf{A}$

- monocoreflective $\Leftrightarrow$ closed under the formation of coproducts and extremal quotients
  - extremal quotient: $G_1 \xrightarrow{q} G_2 \xrightarrow{r} G_3$
  - coproduct: the most general group from $\mathbf{A}$ that is generated by the given groups
Coreflective subcategories in $\mathbf{A}$

- monocoreflective $\iff$ closed under the formation of coproducts and extremal quotients
  - extremal quotient: $G_1 \xrightarrow{q} G_2 \xrightarrow{r} G_3$
  - coproduct: the most general group from $\mathbf{A}$ that is generated by the given groups
- hereditary: closed under the formation of subgroups
Coreflective subcategories in $\mathbf{A}$

- monocoreflective $\iff$ closed under the formation of coproducts and extremal quotients
  - extremal quotient: $G_1 \xrightarrow{q} G_2 \xrightarrow{r} G_3$
  - coproduct: the most general group from $\mathbf{A}$ that is generated by the given groups
- hereditary: closed under the formation of subgroups
- hereditary coreflective $\Rightarrow$ monocoreflective
Coreflective subcategories in $\mathbf{A}$

- monocoreflective $\iff$ closed under the formation of coproducts and extremal quotients
  - extremal quotient: $G_1 \xrightarrow{q} G_2 \xrightarrow{r} G_3$
  - coproduct: the most general group from $\mathbf{A}$ that is generated by the given groups
- hereditary: closed under the formation of subgroups
- hereditary coreflective $\Rightarrow$ monocoreflective
- coreflective, contains $r(\mathbb{Z}) \Rightarrow$ bicoreflective
Coreflective subcategories in $\mathbf{A}$

- monocoreflective $\Leftrightarrow$ closed under the formation of coproducts and extremal quotients
  - extremal quotient: $G_1 \xrightarrow{q} G_2 \xrightarrow{r} G_3$
  - coproduct: the most general group from $\mathbf{A}$ that is generated by the given groups
- hereditary: closed under the formation of subgroups
- hereditary coreflective $\Rightarrow$ monocoreflective
- coreflective, contains $r(\mathbb{Z})$ $\Rightarrow$ bicoreflective
  - e.g. $\mathbf{QTopGr}$ in $\mathbf{STopGr}$, $\mathbf{TopGr}$ in $\mathbf{PTopGr}$
The questions

Which hereditary coreflective subcategories of \( A \) are also bicoreflective in \( A \)?
The questions

Which hereditary coreflective subcategories of $A$ are also bicoreflective in $A$?

- **STopGr, QTopGr**: every hereditary coreflective subcategory of $A$ that contains a non-indiscrete group is bicoreflective in $A$. 

   

What are maximal hereditary coreflective subcategories of $A$ that are not bicoreflective in $A$?

What is the group $r(Z)$?
The questions

Which hereditary coreflective subcategories of $A$ are also bicoreflective in $A$?

- $\text{STopGr}$, $\text{QTopGr}$: every hereditary coreflective subcategory of $A$ that contains a non-indiscrete group is bicoreflective in $A$.
  Are there other epireflective subcategories of $\text{STopGr}$ with this property?
Which hereditary coreflective subcategories of $A$ are also bicoreflective in $A$?

- $\text{STopGr}$, $\text{QTopGr}$: every hereditary coreflective subcategory of $A$ that contains a non-indiscrete group is bicoreflective in $A$. Are there other epireflective subcategories of $\text{STopGr}$ with this property?
- What are maximal hereditary coreflective subcategories of $A$ that are not bicoreflective in $A$?
The questions

Which hereditary coreflective subcategories of \( A \) are also bicoreflective in \( A \)?

- **\( \text{STopGr} \), \( \text{QTopGr} \):** every hereditary coreflective subcategory of \( A \) that contains a non-indiscrete group is bicoreflective in \( A \). Are there other epireflective subcategories of \( \text{STopGr} \) with this property?

- What are maximal hereditary coreflective subcategories of \( A \) that are not bicoreflective in \( A \)?

- What is the group \( r(\mathbb{Z}) \)?
The group $r(\mathbb{Z})$

$r(\mathbb{Z})$ can be:

- A finite cyclic group $\mathbb{Z}_n$
- The group of integers with a topology such that $\mathbb{Z} \to n\mathbb{Z}$ is continuous for every $n \in \mathbb{N}$
- $\mathbb{Z}$ with a topology that is not $T_0$
The group $r(\mathbb{Z})$

$r(\mathbb{Z})$ can be:

- a finite cyclic group $\mathbb{Z}_n$
The group $r(\mathbb{Z})$

$r(\mathbb{Z})$ can be:

- a finite cyclic group $\mathbb{Z}_n$
- the group of integers with a topology such that $\mathbb{Z} \to n\mathbb{Z}$ is continuous for every $n \in \mathbb{N}$
$r(\mathbb{Z})$ can be:

- a finite cyclic group $\mathbb{Z}_n$
- the group of integers with a topology such that $\mathbb{Z} \to n\mathbb{Z}$ is continuous for every $n \in \mathbb{N}$
- $\mathbb{Z}$
The group $r(\mathbb{Z})$

$r(\mathbb{Z})$ can be:

- a finite cyclic group $\mathbb{Z}_n$
- the group of integers with a topology such that $\mathbb{Z} \to n\mathbb{Z}$ is continuous for every $n \in \mathbb{N}$
- $\mathbb{Z}$
- $\mathbb{Z}_n$
The group $r(\mathbb{Z})$

$r(\mathbb{Z})$ can be:

- a finite cyclic group $\mathbb{Z}_n$
- the group of integers with a topology such that $\mathbb{Z} \rightarrow n\mathbb{Z}$ is continuous for every $n \in \mathbb{N}$
- $\mathbb{Z}$
- $\mathbb{Z}_n$
- $\mathbb{Z}$ with a topology that is not $T_0$
The group $r(\mathbb{Z})$

$r(\mathbb{Z})$ can be:

- a finite cyclic group $\mathbb{Z}_n$
- the group of integers with a topology such that $\mathbb{Z} \to n\mathbb{Z}$ is continuous for every $n \in \mathbb{N}$
- $\mathbb{Z}$
- $\mathbb{Z}_n$
- $\mathbb{Z}$ with a topology that is not $T_0$
- $\mathbb{Z}$ with the topology generated by all non-trivial subgroups
STopGr, QTopGr:
every hereditary coreflective subcategory of A that contains a non-indiscrete group is bicoreflective in A
**STopGr, QTopGr:**
every hereditary coreflective subcategory of $A$ that contains a non-indiscrete group is bireflective in $A$

ereditary coreflective, not bireflective:
  - only the trivial group
  - all indiscrete groups
$r(\mathbb{Z}) = \mathbb{Z}$

- **STopGr, QTopGr:**
  every hereditary coreflective subcategory of $A$ that contains a non-indiscrete group is bicoreflective in $A$

- **hereditary coreflective, not bicoreflective:**
  - only the trivial group
  - all indiscrete groups

- **A:** extremal epireflective in $STopGr$, $A \subseteq STopAb$
\( r(\mathbb{Z}) = \mathbb{Z} \)

- **STopGr, QTopGr:**
  every hereditary coreflective subcategory of \( A \) that contains a non-indiscrete group is bicoreflective in \( A \)
  hereditary coreflective, not bicoreflective:
  - only the trivial group
  - all indiscrete groups

- **A:** extremal epireflective in **STopGr**, \( A \subseteq \text{STopAb} \)
  **B:** such groups \( G \) from \( A \) that no infinite cyclic subgroup of \( G \) is \( T_0 \)
**STopGr, QTopGr:**
every hereditary coreflective subcategory of \( A \) that contains a non-indiscrete group is bicoreflective in \( A \)
hereditary coreflective, not bicoreflective:
- only the trivial group
- all indiscrete groups

**A:** extremal epireflective in \( STopGr, \ A \subseteq STopAb \)
**B:** such groups \( G \) from \( A \) that no infinite cyclic subgroup of \( G \) is \( T_0 \)
**B** is the largest hereditary coreflective subcategory of \( A \) that is not bicoreflective in \( A \)
$r(\mathbb{Z}) = \mathbb{Z}_n$

- **A:** such groups $G$ that every element of $G$ is a divisor of $n$
\[ r(\mathbb{Z}) = \mathbb{Z}_n \]

- **A**: such groups \( G \) that every element of \( G \) is a divisor of \( n \)
  - \( A \) is extremal epireflective in \( \text{STopGr} \), \( r(\mathbb{Z}) = \mathbb{Z}_n \)
$r(\mathbb{Z}) = \mathbb{Z}_n$

- **A**: such groups $G$ that every element of $G$ is a divisor of $n$

  A is extremal epireflective in $\text{STopGr}$, $r(\mathbb{Z}) = \mathbb{Z}_n$

  every hereditary coreflective subcategory of $A$ that contains a non-indiscrete group is bicoreflective in $A$
\( r(\mathbb{Z}) = \mathbb{Z}_n \)

- **A**: such groups \( G \) that every element of \( G \) is a divisor of \( n \)  
  \( A \) is extremal epireflective in \( \text{STopGr} \),  
  every hereditary coreflective subcategory of \( A \) that contains a non-indiscrete group is bicoreflective in \( A \)
- \( r(\mathbb{Z}) = \mathbb{Z}_p \), \( p \) is a prime number
$r(\mathbb{Z}) = \mathbb{Z}_n$

- **A**: such groups $G$ that every element of $G$ is a divisor of $n$
  
  $A$ is extremal epireflective in $\text{STopGr}$, $r(\mathbb{Z}) = \mathbb{Z}_n$
  
  every hereditary coreflective subcategory of $A$ that contains a non-indiscrete group is bicoreflective in $A$

- $r(\mathbb{Z}) = \mathbb{Z}_p$, $p$ is a prime number
  
  every hereditary coreflective subcategory of $A$ that contains a non-indiscrete group is bicoreflective in $A$
A: such groups $G$ that every element of $G$ is a divisor of $n$

$A$ is extremal epireflective in $\text{STopGr}$, $r(\mathbb{Z}) = \mathbb{Z}_n$

every hereditary coreflective subcategory of $A$ that contains a
non-indiscrete group is bicoreflective in $A$

$r(\mathbb{Z}) = \mathbb{Z}_p$, $p$ is a prime number

every hereditary coreflective subcategory of $A$ that contains a
non-indiscrete group is bicoreflective in $A$

$A \subseteq \text{STopAb}$, $r(\mathbb{Z}) = \mathbb{Z}_n$, $n = p_1^{\alpha_1} \cdots p_k^{\alpha_k}$
\( r(\mathbb{Z}) = \mathbb{Z}_n \)

- \( \textbf{A:} \) such groups \( G \) that every element of \( G \) is a divisor of \( n \)
  - \( \textbf{A} \) is extremal epireflective in \( \text{STopGr} \), \( r(\mathbb{Z}) = \mathbb{Z}_n \)
  - every hereditary coreflective subcategory of \( A \) that contains a non-indiscrete group is bicoreflective in \( A \)

- \( r(\mathbb{Z}) = \mathbb{Z}_p \), \( p \) is a prime number
  - every hereditary coreflective subcategory of \( A \) that contains a non-indiscrete group is bicoreflective in \( A \)

- \( \textbf{A} \subseteq \text{STopAb}, \ r(\mathbb{Z}) = \mathbb{Z}_n, \ n = p_1^{\alpha_1} \cdots p_k^{\alpha_k} \)

\( \textbf{B_i:} \) such groups \( G \) from \( \textbf{A} \) that if \( H \) is a cyclic subgroup of \( G \) of order \( p_i^{\alpha_i} \) then its topology is strictly coarser then the subspace topology induced from \( r(\mathbb{Z}) \)
\( r(\mathbb{Z}) = \mathbb{Z}_n \)

- **A**: such groups \( G \) that every element of \( G \) is a divisor of \( n \) is extremal epireflective in \( \text{STopGr} \), \( r(\mathbb{Z}) = \mathbb{Z}_n \) every hereditary coreflective subcategory of \( A \) that contains a non-indiscrete group is bicoreflective in \( A \)

- \( r(\mathbb{Z}) = \mathbb{Z}_p \), \( p \) is a prime number every hereditary coreflective subcategory of \( A \) that contains a non-indiscrete group is bicoreflective in \( A \)

- \( A \subseteq \text{STopAb} \), \( r(\mathbb{Z}) = \mathbb{Z}_n \), \( n = p_1^{\alpha_1} \cdots p_k^{\alpha_k} \)

  - \( B_i \): such groups \( G \) from \( A \) that if \( H \) is a cyclic subgroup of \( G \) of order \( p_i^{\alpha_i} \) then its topology is strictly coarser than the subspace topology induced from \( r(\mathbb{Z}) \)

  - \( B_i \) are maximal hereditary coreflective subcategories of \( A \) that are not bicoreflective in \( A \)
$r(\mathbb{Z}) = \mathbb{Z}$, topology is not $T_0$

- $A \subseteq \text{STopAb}$
  - the closure of $\{0\}$ in $r(\mathbb{Z})$ is $\langle n \rangle$
\( r(\mathbb{Z}) = \mathbb{Z} \), topology is not \( T_0 \)

- \( A \subseteq \text{STopAb} \)
  - the closure of \{0\} in \( r(\mathbb{Z}) \) is \( \langle n \rangle \)

- \( \langle n \rangle \rightarrow r(\mathbb{Z}) \) is an epimorphism: every hereditary coreflective subcategory of \( A \) that contains a non-trivial group is bicoreflective in \( A \)
$r(\mathbb{Z}) = \mathbb{Z}$, topology is not $T_0$

- $A \subseteq \text{STopAb}$
  the closure of $\{0\}$ in $r(\mathbb{Z})$ is $\langle n \rangle$

- $\langle n \rangle \rightarrow r(\mathbb{Z})$ is an epimorphism: every hereditary coreflective subcategory of $A$ that contains a non-trivial group is bicoreflective in $A$

- $k \mid n$ is minimal such that $\langle k \rangle \rightarrow r(\mathbb{Z})$ is an not epimorphism:
A \subseteq S\text{TopAb} \\
the closure of \{0\} in \( r(\mathbb{Z}) \) is \( \langle n \rangle \) \\
\( \langle n \rangle \rightarrow r(\mathbb{Z}) \) is an epimorphism: every hereditary coreflective subcategory of A that contains a non-trivial group is bicoreflective in A \\
\( k | n \) is minimal such that \( \langle k \rangle \rightarrow r(\mathbb{Z}) \) is an not epimorphism: 

\( B_k \): such groups \( G \) from A that if \( H \) is an infinite cyclic subgroup of \( G \) then the topology of \( H \) is not finer that the topology of \( \langle k \rangle \)
$r(\mathbb{Z}) = \mathbb{Z}$, topology is not $T_0$

- $A \subseteq \text{STopAb}$
  - the closure of $\{0\}$ in $r(\mathbb{Z})$ is $\langle n \rangle$
- $\langle n \rangle \rightarrow r(\mathbb{Z})$ is an epimorphism: every hereditary coreflective subcategory of $A$ that contains a non-trivial group is bicoreflective in $A$
- $k|n$ is minimal such that $\langle k \rangle \rightarrow r(\mathbb{Z})$ is an not epimorphism: $B_k$: such groups $G$ from $A$ that if $H$ is an infinite cyclic subgroup of $G$ then the topology of $H$ is not finer that the topology of $\langle k \rangle$
  - $B_k$ are maximal hereditary coreflective subcategories of $A$ that are not bicoreflective in $A$
$r(\mathbb{Z}) = \mathbb{Z}$, topology is generated by all non-trivial subgroups

- $A \subseteq \text{TopAb}$
$r(\mathbb{Z}) = \mathbb{Z}$, topology is generated by all non-trivial subgroups $A \subseteq \text{TopAb}$, where $B_p (p - \text{prime number})$:
\( r(\mathbb{Z}) = \mathbb{Z} \), topology is generated by all non-trivial subgroups

- \( A \subseteq \text{TopAb} \)
  - \( B_p \) (\( p \) – prime number): such groups \( G \) from \( A \) that if \( H \) is an infinite cyclic subgroup of \( G \) then there exists \( n \in \mathbb{N} \) such that the subgroup of index \( p^n \) is not open in \( H \)
$r(\mathbb{Z}) = \mathbb{Z}$, topology is generated by all non-trivial subgroups

- $\mathbb{A} \subseteq \text{TopAb}$
- $\mathbb{B}_p$ ($p$ – prime number): such groups $G$ from $\mathbb{A}$ that if $H$ is an infinite cyclic subgroup of $G$ then there exists $n \in \mathbb{N}$ such that the subgroup of index $p^n$ is not open in $H$.
- $\mathbb{B}_p$ are maximal hereditary coreflective subcategories of $\mathbb{A}$ that are not bicoreflective in $\mathbb{A}$.
Suggestions for further research

- What happens in the case of non-abelian groups?
Suggestions for further research

- What happens in the case of non-abelian groups?
- What happens when $r(\mathbb{Z})$ is the group of integers with a topology generated by some of its non-trivial subgroups?
Suggestions for further research

- What happens in the case of non-abelian groups?
- What happens when \( r(\mathbb{Z}) \) is the group of integers with a topology generated by some of its non-trivial subgroups?
- What happens when \( r(\mathbb{Z}) \) is the group of integers with a topology that is not generated by its subgroups?
Thank you for your attention.