

POLARISED PARTITION RELATIONS FOR ORDER TYPES

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ABSTRACT. We analyse partitions of products with two ordered factors in two classes where both factors are countable or well-ordered and at least one of them is countable. This relates the partition properties of these products to cardinal characteristics of the continuum. We build on work by Erdős, Garti, Jones, Orr, Rado, Shelah and Szemerédi. In particular, we show that a theorem of Jones extends from the natural numbers to the rational ones but consistently extends only to three further equimorphism classes of countable orderings. This is made possible by applying a thirteen-year old theorem of Orr about embedding a given order into a sum of finite orders indexed over the given order.

This is joint work with Lukas Klausner, a preprint is available at <http://www.logic.univie.ac.at/~weinertt92/return.pdf>.

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