ŠÁRKA STEJSKALOVÁ. The indestructibility of the tree property.
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We show that the tree property at $\omega_2$ in the Mitchell model $V[M]$ is indestructible
by all ccc forcing notions which live in an intermediate model $V[\text{Add}(\omega, \kappa)] \subseteq V[M]$,
provided we start with a supercompact cardinal $\kappa$ ($\kappa$ becomes $\omega_2$). This shows that it
is consistent that a large class of ccc forcings cannot add new $\omega_2$-Aronszajn trees (for
instance, consistently, no ccc forcing living in $L$ adds an $\omega_2$-Aronszajn tree). With a
fancier forcing, this result extends to all forcings which are (i) $\omega_1$-closed and $\omega_2$-cc and
(ii) $\omega_2$-directed closed. The result generalizes to cardinals larger than $\omega_2$ and allows
applications to Prikry-style forcing notions.

The work is joint with R. Honzik.