Quotients of free topological groups

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Let, as usual, $F(X)$ and $A(X)$ denote the free topological group and the free abelian topological group of a Tychonoff space X, respectively. $A(X)$ is a natural quotient group of $F(X)$, and for every X there is a quotient homomorphism from $A(X)$ onto the group of integers $\mathbb{Z}$.

A topological space X is called *ω-bounded* if the closure of every countable subset of X is compact. Clearly, every compact space is ω-bounded, while every ω-bounded space is countably compact.

Proposition 1. *Let X be a non-scattered ω-bounded Tychonoff space. Then both $A(X)$ and $F(X)$ admit an open continuous homomorphism onto the circle group $\mathbb{T}$.*

Theorem 2. *Let X be an ω-bounded Tychonoff space. Then the following conditions are equivalent: (a) X is scattered; (b) Every metrizable quotient group of $F(X)$ or $A(X)$ is discrete and finitely generated.*

Corollary 3. *Let X be either the compact space of ordinals $[0, \alpha]$ with the order topology or the one-point compactification of an arbitrary discrete space. Then every metrizable quotient group of $F(X)$ or $A(X)$ is discrete and finitely generated.*

Theorem 4. *Let X be a Tychonoff space satisfying the following conditions: (1) the closure of every countable subset of X is countable and compact; (2) every countable compact subset of X is a retract of X. Then every separable quotient group of $F(X)$ or $A(X)$ is countable.*

Corollary 5. *Let X be either the space of ordinals $[0, \alpha)$ with the order topology or the one-point compactification of an arbitrary discrete space. Then every separable quotient group of $F(X)$ or $A(X)$ is countable.*

This is joint work with Michael Tkachenko.