On sets meeting every line in a set of measure 1

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One can ask whether there exist a subset $S$ of the plane having a prescribed size on each line, for example it is known that there is a set containing exactly two points from each line.

We heard the following similar question from A. Kumar (but others had asked it before): Does there exist a set on the plane meeting each line in a ($\lambda^1$-measurable) set of measure 1? Under $CH$ it is not difficult to construct such a set. On the other hand it is proved (in $ZFC$) that a set of this type cannot be $\lambda^2$-measurable [1]. It remained open whether such a set always exists, or its non-existence is consistent with $ZFC$.

Using the theory of harmonic functions, and a consistent inequality between two cardinal invariants of the null ideal we proved that consistently, there is no set on the plane intersecting each line in a set of (1-dimensional) measure 1. This is a joint work with Márton Elekes and Zoltán Vidnyánszky.

References