

Open colorings

Let X be a topological space. We say that $OCA(X)$ holds if and only if for every open partition $[X]^2 = K_0 \cup K_1$, either:

1. There exists an uncountable $H \subseteq X$ such that $[H]^2 \subseteq K_0$ (0-homogeneous), or else
2. There is a family $\langle H_n : n \in \omega \rangle$ such that $X = \bigcup_{n \in \omega} H_n$ and $[H_n]^2 \subseteq K_1$ for every $n \in \omega$ (σ -1-homogeneous).

This statement is due to Todorćević and holds for the real line. In 80's, Todorćević showed that it is relative consistent with ZFC that: $OCA(X)$ holds for every subspace X of the real line (OCA). In order to generalize this statement, Todorćević conjectured that the following is relative consistent with ZFC: If X is a regular space with no uncountable discrete subspace, then $OCA(X)$ holds. I will talk about the situation of $OCA(X)$ for the Sorgenfrey line and their subspaces.