LARGE CARDINALS IN GENERAL TOPOLOGY III

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Winter School in Abstract Analysis
Hejnice, January 26–February 2, 2019

Locally presentable categories are precisely those categories which can be axiomatized by limit sentences in an infinitary first-order logic.

They include varieties and quasivarieties of algebras.

Definition

Let $X = \varprojlim\{X_i, \pi_{i,j}\}_I$. A map $f : X \to A$ is said to depend on a (down-directed) set $J \subset I$ if there exists a map $g : \varprojlim\{\overline{\mathrm{pr}_i(X_i)}, \pi_{i,j}\}_J \to A$ in $\mathcal{C}$ such that $f = g\, p_J$.

Task

Let $\mathcal{C}$ be generated by a space A. We are looking for a cardinal λ such that every map from $\varprojlim\{X_i, \pi_{i,j}\}_I$ into A depends on some $J \subset I$ with $|J| < \lambda$.

Such a least cardinal λ is denoted by $\mathrm{coord}(\mathcal{C}, A)$.

Special task

When discrete ordered index sets I are used only, notations $\mathrm{coord}_d(\mathcal{C}, A)$ are used.

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When discrete ordered index sets I are used only, notations $\mathrm{coord}_d(\mathcal{C}, A)$ are used.

Theorem

Let $X = \varprojlim\{X_i, \pi_{i,j}\}_I$ *and* $f : X \to A$, J *be a down-directed subset of* I. *Then* f *depends on* J *iff* f *depends on* J *regarding* X *as a subspace of* $\prod_I X_i$ *and the factorized map extends in* $\mathcal{C}$ *onto* $\overline{\mathrm{pr}_J(X)}$.

Theorem

$\mathrm{coord}(\mathcal{C}, A) \leq \lambda$ *iff for any family* $\{X_i\}_I \subset \mathcal{C}$, *any closed subset* $X \subset \prod_I X_i$ *and any map* $f : X \to A$, *the map* f *depends on some* $J \subset I$ *with* $|J| < \lambda$ *and the factorized map can be extended to a continuous map* $\overline{\mathrm{pr}_J(X)} \to A$.

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$\mathrm{coord}_d(\mathcal{C}, A) \leq \lambda$ *iff for any family* $\{X_i\}_I \subset \mathcal{C}$ *and any map* $f : \prod_I X_i \to A$, *the map* f *factorizes in* $\mathcal{C}$ *via a subproduct* $\prod_J X_i$ *for some* $J \subset I$ *with* $|J| < \lambda$.

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Let $\mathfrak{m}_{\mathcal{C}}$ be the least cardinal κ such that a discrete space of cardinality κ does not belong to $\mathcal{C}$. In fact,
$\mathfrak{m}_{\mathcal{C}} = \min\{\kappa; \text{there exists } X \in \mathcal{C} \text{ that is not pseudo-}\kappa\text{-compact}\}$

The cardinal $\mathfrak{m}_{\mathcal{C}}$ is measurable or equals to ∞.
Always $\mathrm{coord}(\mathcal{C}, A) \geq \mathrm{coord}_d(\mathcal{C}, A) \geq \mathfrak{m}_{\mathcal{C}}$
If $\mathcal{C}$ is simple and $\mathfrak{m}_{\mathcal{C}} < \infty$ then $\mathrm{coord}_d(\mathcal{C}) < \infty$.

$\mathfrak{m}_{\mathcal{C}} > \omega$

If $\mathfrak{m}_0 < \mathfrak{m}_{\mathcal{C}} < \infty$ then $\mathrm{coord}_d(\mathcal{C}, A) = \mathfrak{m}_{\mathcal{C}}$ provided $\overline{\psi}(\Delta_A) < \mathfrak{m}_{\mathcal{C}}$.
Thus $\mathrm{coord}_d(\texttt{RealComp}) = \mathfrak{m}_1$.

Let $\mathfrak{m}_{\mathcal{C}}$ be the least cardinal κ such that a discrete space of cardinality κ does not belong to $\mathcal{C}$. In fact,
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Theorem (N.Noble, M.Ulmer 1972)

Let κ be a regular cardinal. If $\prod_I X_i$ is pseudo-κ-compact then every continuous $f : \prod_I X_i \to A$ depends on less than κ coordinates for any A with $\overline{\psi}(\Delta_A) < \kappa$.
If all X_i are completely regular and $\prod_I X_i$ is not pseudo-κ-compact, where $\mathrm{cof}(\kappa) > \omega$, then there exists a continuous real-valued function on the product not depending on less than κ coordinates

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Definition

A regular cardinal κ is said to be λ-strongly compact for $\omega < \lambda \leq \kappa$ if every κ-complete filter on any set has an extension to λ-complete ultrafilter.

Every λ-strongly compact cardinal is μ-strongly compact for any infinite $\mu \leq \mathfrak{m}(\lambda)$.

Every λ-strongly compact cardinal is measurable and the first uncountable measurable cardinal $\mathfrak{m}_1$ may be ω_1-strongly compact.

If κ is a λ-strongly compact cardinal then $\upsilon_\lambda(\mu) \cap U_\mu \neq \emptyset$ for every μ with $\mathrm{cof}(\mu) \geq \kappa$.

In particular, if $\mathfrak{m}_1$ is ω_1-strongly compact then $\upsilon(\mu) \cap U_\mu \neq \emptyset$ for every μ with $\mathrm{cof}(\mu) \geq \mathfrak{m}_1$ (clearly, $\upsilon(\mu) \cap U_\mu = \emptyset$ if $\mathrm{cof}(\mu) < \mathfrak{m}_1$).

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General limits, realcompact spaces

Theorem

Let μ-strongly compact cardinals exist and κ be the smallest one. Let A be such a space that every μ-complete ultrafilter on any set D converges in a reflection $r_A(D)$ of D in A-compact spaces. Then $\operatorname{coord}(\mathcal{C}(A), A) \leq \max(\kappa, \chi(A)^+)$.

Theorem (Realcompact spaces)

If ω_1-strongly compact cardinals exists and λ is the smallest one then for $\mathcal{C}$ composed of realcompact spaces and containing $\mathbb{N}$ one has $\mathfrak{m}_1 \leq \operatorname{coord}(\mathcal{C}) \leq \lambda$.

Corollary

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Let μ-strongly compact cardinals exist and κ be the smallest one. Let A be such a space that every μ-complete ultrafilter on any set D converges in a reflection $r_A(D)$ of D in A-compact spaces, and let $\chi(A) < \kappa$. Then $\mathrm{coord}(\mathcal{C}(A), A) \leq \kappa$.

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1. Let $f : X \to A, X$ is a closed subspace of A^I, does not depend on less than κ coordinates. Then for every $J \in [I]^{<\kappa}$ the following sets are nonempty:

$$C_J = \{x \in X;\ \text{there is } y \in X \text{ such that } \operatorname{pr}_J(x) = \operatorname{pr}_J(y), f(x) \neq f(y)\}.$$

The sets C_J form a base of a κ-complete filter $\mathcal{F}$ in the set X. By our assumption, $\mathcal{F}$ can be extended to a μ-complete ultrafilter on the set X and it has an accumulation point ξ in the space X.

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Theorem

Let μ-strongly compact cardinals exist and κ be the smallest one. Let A be such a space that every μ-complete ultrafilter on any set D converges in a reflection $r_A(D)$ of D in A-compact spaces, and let $\chi(A) < \kappa$. Then $\operatorname{coord}(\mathcal{C}(A), A) \leq \kappa$.

2. Now f depends on some $J_0 \in [I]^{<\kappa}$ and $f = f_J \mathrm{pr}_J$ for any $J \in \mathcal{J} = \{J \in [I]^{<\kappa}, J \supset J_0\}$. Assume no such f_J is continuous and denote

$$C_J = \{x \in X; f_J \text{ is not continuous at } \mathrm{pr}_J(x)\}.$$

Again, the sets C_J form a base of a κ-complete filter $\mathcal{F}$ in the set X extendible to a μ-complete ultrafilter on the set X and, thus, having an accumulation point in X.

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3. Now f depends continuously on some $J_0 \in [I]^{<\kappa}$. Denote again $\mathcal{J} = \{J \in [I]^{<\kappa}, J \supset J_0\}$. Assume no such f_J can be continuously extended to $\overline{\operatorname{pr}_J(X)}$. Then we can define

$$C_J = \{x \in \operatorname{pr}_J^{-1}(\overline{\operatorname{pr}_J(X)}); f_J \text{ does not extend continuously to } \operatorname{pr}_J(x)\}.$$

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Herrlich's κ-compact spaces

Let κ be an infinite cardinal. The class of κ-compact spaces is denoted by $\mathcal{H}(\kappa)$.

We repeat that a Tikhonov space is κ-compact if every maximal zero-filter, that is κ-complete, is fixed; $[0,1]^\kappa \setminus \{1\}$-compact spaces are exactly κ^+-compact spaces.

If κ is not measurable then $\mathfrak{m}_{\mathcal{H}(\kappa)} = \mathfrak{m}(\kappa)$

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$\mathfrak{m}(\kappa) \leq \mathrm{coord}(\mathcal{H}(\kappa)) \leq \inf\{\kappa; \kappa$ *is* κ*-strongly compact*$\}$.

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Dieudonné complete spaces

Dieudonné complete space is a topological space induced by a complete uniformity.

The class `Dieud` of Dieudonné complete spaces is simple iff the class of measurable cardinals is a set (the class is generated by $\{H(\mathfrak{m})\}$), $\mathfrak{m}$ measurable).

By $\texttt{Dieud}_\kappa$ we denote the class of Dieudonné complete spaces X generated by $\{H(\lambda); \lambda < \kappa\}$ (i.e., every closed discrete set in X is of cardinality less than $\mathfrak{m}(\kappa)$).

Theorem

1. $\mathrm{coord}_d(\texttt{Dieud}) = \infty$.
2. $\mathrm{coord}_d(\texttt{Dieud}_\kappa) = \mathfrak{m}(\kappa)$
3. $\mathfrak{m}(\kappa) \leq \mathrm{coord}(\texttt{Dieud}_\kappa) \leq \inf\{\mu; \mu \textit{ is } \mathfrak{m}(\kappa)\textit{-strongly compact}\}$.

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Uniform spaces

Theorem (Viddossich, 1970)

Every uniformly continuous map from a subspace of a product of uniform spaces into a uniform space A depends on at most $w_u(A)$ many coordinates and the factorized map is uniformly continuous.

Theorem

Let $\mathcal{C}$ be a productive and closed hereditary subcategory of $\texttt{Unif}_2$ *with generators $\mathcal{A}$. Then $\operatorname{coord}_d(\mathcal{C},\mathcal{A}) \le (\sup\{w_u(A); A \in \mathcal{A}\})^+$. If $\mathcal{A}$ consist of complete spaces then $\operatorname{coord}(\mathcal{C},\mathcal{A}) \le (\sup\{w_u(A); A \in \mathcal{A}\})^+$.*

Corollary

1. *For every simple subcategory $\mathcal{C}$ of* $\texttt{Unif}_2$ *one has $\operatorname{coord}_d(\mathcal{C}) < \infty$.*
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Uniform spaces

Theorem (Viddossich, 1970)

Every uniformly continuous map from a subspace of a product of uniform spaces into a uniform space A depends on at most $w_u(A)$ many coordinates and the factorized map is uniformly continuous.

Theorem

Let $\mathcal{C}$ be a productive and closed hereditary subcategory of $\mathtt{Unif}_2$ *with generators $\mathcal{A}$. Then* $\operatorname{coord}_d(\mathcal{C}, \mathcal{A}) \leq (\sup\{w_u(A); A \in \mathcal{A}\})^+$. *If $\mathcal{A}$ consist of complete spaces then* $\operatorname{coord}(\mathcal{C}, \mathcal{A}) \leq (\sup\{w_u(A); A \in \mathcal{A}\})^+$.

Corollary

1. *For every simple subcategory $\mathcal{C}$ of* $\mathtt{Unif}_2$ *one has* $\operatorname{coord}_d(\mathcal{C}) < \infty$.
2. *If $\mathcal{C}$ is the class of all complete uniform spaces then* $\operatorname{coord}(\mathcal{C}) = \omega_1$.

Precompact spaces

Consider the category Prec of all precompact (totally bounded) uniform Hausdorff spaces. It has an interesting class of generators, namely $P_\kappa = [0,1]^\kappa \setminus \{1\}$ for all infinite cardinals κ.

Theorem

$\mathrm{coord}_d(\mathtt{Prec}, \{P_\kappa\}) = \infty$.

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$$\kappa \leq \mathrm{coord}(\mathtt{Prec}(P_\kappa), P_\kappa) \leq \inf\{\lambda; \lambda \textit{ is a } \kappa\textit{-strongly compact cardinal}\}.$$

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Problems

Is $\mathcal{C}_{\mathfrak{m}} = \mathcal{U}_{\mathfrak{m}}$ for measurable cardinals $\mathfrak{m}$? (Yes, if $\mathfrak{m}$ is $\mathfrak{m}$-strongly compact.)

Are the classes $\mathcal{U}_{\kappa}$ simple? (Not, if $\mathfrak{m}_1$ does not exists.)

Is the class of all zerodimensional realcompact spaces simple? (Not, if $\mathfrak{m}_1$ does not exists.)

Find a topological characterizatiuon of the property $\upsilon(X \times Y) = \upsilon(X) \times \upsilon(Y)$.

Is there a nontrivial productive class in Top closed under quotients and disjoint sums? (Not, if $\mathfrak{s}$ does not exists.)

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