ON THE JOSEFSON–NISSENZWEIG THEOREM FOR $C(K)$-SPACES

DAMIAN SOBOTA

The celebrated Josefson–Nissenzweig theorem asserts that for every infinite-dimensional Banach space X there exists a sequence of continuous functionals (φ_n) on X such that $\|\varphi_n\| = 1$ and $\varphi_n(x) \underset{n}{\longrightarrow} 0$ for every $x \in X$. In the special case of the Banach space $C(K)$ of continuous real-valued functions on a compact Hausdorff space K the theorem states that there exists a sequence of signed regular Borel measures (μ_n) of variation 1 such that $\int_K f \mathrm{d}\mu_n \underset{n}{\longrightarrow} 0$ for every $f \in C(K)$. The original proofs of Josefson and Nissenzweig (and those following them) were highly non-trivial and, most of all, non-constructive; the only (partially) constructive proof of the theorem known so far is due to Behrends, however — in the case of $C(K)$-spaces — the measures μ_n obtained in his proof are defined using so-called Banach limits and thus quite complicated. Since the theorem is extremely useful in the study of $C(K)$-spaces, we were motivated to find more simple proofs of it (at least in some important cases).

During my talk I will show that many compact spaces admit Josefson–Nissenzweig sequences of measures being just finite linear combinations of point-measures (Dirac's deltas). To the class of such compact spaces belong among other spaces with non-trivial convergent sequences, Efimov spaces obtained by minimal extensions, and products. I will also show that admitting such a Josefson–Nissenzweig sequence is strongly related to the so-called Grothendieck property of $C(K)$-spaces.

This is a joint work with Lyubomyr Zdomskyy.

PS. No knowledge of Banach space theory is required to understand the talk.

Kurt Gödel Research Center for Mathematical Logic, University of Vienna
E-mail address: ein.damian.sobota@gmail.com