SCATTERED COMPACT SPACES ARE ATTRACTORS OF GENERALIZED ITERATED FUNCTION SYSTEMS

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Abstract. R. Miculescu and A. Mihail, see [MM1], [MM2], introduced a concept of a generalized iterated function system (GIFS in short), a particular extension of a classical IFS. The idea is that, instead of families of selfmaps of a metric space \( X \), GIFSs consist of maps defined on a finite Cartesian product \( X^m \) with values in \( X \). It turned out that a great part of the classical Hutchinson–Barnsley theory has natural counterpart in this GIFSs’ case. On the other hand, there are known only few examples of fractal sets which are generated by GIFSs, but which are not IFSs’ fractals.

During the talk I will show that each compact scattered metrizable space \( X \) is homeomorphic to the attractor of some GIFS on the real line. This fully distinguishes the class of GIFSs’ fractals from the class of IFSs’ fractals, since (as was proved by M. Nowak in [N]), if the Cantor-Bendixon height of \( X \) is limit ordinal, then \( X \) is not homeomorphic to attractor of any IFS, i.e., it is not a topological IFS fractal.

Moreover, I will show that there are compact scattered subsets of the real line which distinguish (in metric sense) certain classes of GIFSs' fractals.

References


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