CICHOŃ'S MAXIMUM

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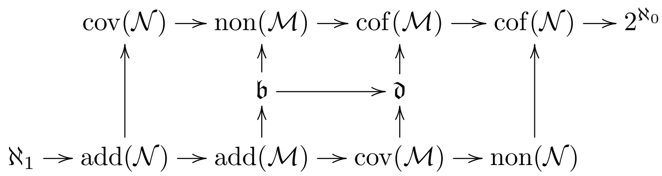
Background

For any ideal I on a set X, we write $\mathrm{add}(I)$, $\mathrm{cov}(I)$, $\mathrm{non}(I)$, for the answers to the questions

- How many ideal sets do you need to **add up**, to get a non-ideal set?
- How many ideal sets do you need to **cover** all of X?
- How many points of X do you need to get a **non**-ideal set?

and we write $\mathrm{cof}(I)$ for the **cofinality** of I, the smallest cardinality of a set that is cofinal in the partial order $(I, \subseteq)$.

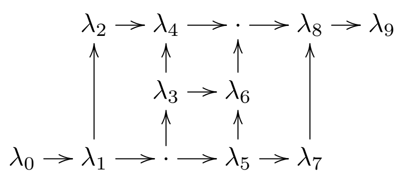
Cichoń's diagram collects those cardinals for the following ideals $\mathcal{N}$ (the ideal of Lebesgue null sets) and $\mathcal{M}$ (the ideal of meager (=first category) sets), as well as the numbers $\mathfrak{b}$ and $\mathfrak{d}$ (the unbounding number and the dominating number, or equivalently the additivity number and covering number of the σ-ideal generated by the compact sets of irrationals), and the numbers $\aleph_1$ and $\mathfrak{c} = 2^{\aleph_0}$ (equivalently, the additivity and covering number of the ideal of countable sets).



An arrow between $\mathfrak{x}$ and $\mathfrak{y}$ indicates that ZFC proves $\mathfrak{x} \leq \mathfrak{y}$. ZFC proves $\mathrm{add}(\mathcal{M}) = \min(\mathfrak{b}, \mathrm{cov}(\mathcal{M}))$ and $\mathrm{cof}(\mathcal{M}) = \max(\mathfrak{d}, \mathrm{non}(\mathcal{M}))$, so at most 10 different values can appear in this diagram.

Theorem

In a recent paper with Jakob Kellner and Saharon Shelah we constructed (using 4 strongly compact cardinals) a ZFC universe where 10 of the cardinals in Cichoń's diagram have distinct values: $\aleph_1 < \mathrm{add}(\mathcal{N}) < \mathrm{cov}(\mathcal{N}) < \mathfrak{b} < \mathrm{non}(\mathcal{M}) < \mathrm{cov}(\mathcal{M}) < \mathfrak{d} < \mathrm{non}(\mathcal{N}) < \mathrm{cof}(\mathcal{N}) < 2^{\aleph_0}$.



Proof

We first use a construction from a previous paper (with Mejía and Shelah) to get a partial order P forcing different values on the left side of the diagram, and then use a *Boolean ultrapower* of P to also increase the cardinals on the right hand side of the diagram (while keeping the values we have already determined on the left hand side).

In my talk I will sketch some interesting fragments of this construction.

Links

- The left side of Cichoń's Diagram: https://arxiv.org/abs/1504.04192, PAMS 144 (2016)
- Cichoń's maximum: https://arxiv.org/abs/1708.03691, submitted.