BLOCK SEQUENCES WITH PROJECTIONS INTO A SEQUENCE OF HAPPY FAMILIES

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Definition 1. Let $k \geq 1$. Fix $P_{\min}, P_{\max} \subseteq \{1, \dots, k\}$. Let $PP = \{(i, x) \,:\, x \in \{\min, \max\}, i \in P_x\}$ and let

$$\bar{\mathcal{R}} = \{(\iota, \mathcal{R}_\iota) \,:\, \iota \in PP\}$$

be a PP-sequence of pairwise non-nearly coherent Ramsey ultrafilters.

We let $(\mathrm{FIN}_k)^\omega(\bar{\mathcal{R}})$ denote the set of FIN_k-blocksequences $\bar{a}$ with the following properties:

$$(\forall i \in P_{\min})\{\min(a_n^{-1}[\{i\}]) \,:\, n \in \omega\} \in \mathcal{R}_{i,\min} \wedge$$
$$(\forall i \in P_{\max})\{\max(a_n^{-1}[\{i\}]) \,:\, n \in \omega\} \in \mathcal{R}_{i,\max}.$$

Definition 2. The Tetris-finite-union closure of an element $\bar{a} \in (\mathrm{FIN}_k)^\omega(\bar{\mathcal{R}})$ is the following set

$$\begin{aligned}\mathrm{TFU}_k(\bar{a}) = &\{T^{(j_0)}(a_{n_0}) + \dots + T^{(j_\ell)}(a_{n_\ell}) \,:\\ &\ell \in \omega \smallsetminus \{0\}, n_0 < \dots < n_\ell, j_i \in k, \exists r j_r = 0\}\end{aligned}$$

For $\bar{a}, \bar{b} \in (\mathrm{FIN}_k)^\omega$ we let $\bar{b} \sqsubseteq_k \bar{a}$ if $\bar{b} \subseteq \mathrm{TFU}_k(\bar{a})$.

We sketch a proof of a common strengthening of a Theorem of Blass for $k = 1$, $PP = \{(1, \min), (1, \max)\}$ and Gowers for $k \geq 1$, $PP = \emptyset$:

Theorem 3. *Let $\bar{a} \in (\mathrm{FIN}_k)^\omega(\bar{\mathcal{R}})$ and let c be a colouring of $\mathrm{TFU}_k(\bar{a})$ into finitely many colours. Then there is a $\bar{b} \sqsubseteq_k \bar{a}$, $\bar{b} \in (\mathrm{FIN}_k)^\omega(\bar{\mathcal{R}})$ such that $\mathrm{TFU}_k(\bar{b})$ is c-monochromatic.*

This theorem gives rise to a new proper non-σ-centred forcing with the pure decision property.

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