BLOCK SEQUENCES WITH PROJECTIONS INTO A SEQUENCE OF HAPPY FAMILIES

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Definition 1. Let $k \geq 1$. Fix $P_{\min}, P_{\max} \subseteq \{1, \ldots, k\}$. Let $PP = \{(i, x) : x \in \{\min, \max\}, i \in P_x\}$ and let
\[
\bar{R} = \{(i, R_i) : i \in PP\}
\]
be a $PP$-sequence of pairwise non-nearly coherent Ramsey ultrafilters.

We let $(\text{FIN}_k)^{\omega}(\bar{R})$ denote the set of $\text{FIN}_k$-blocksequences $\bar{a}$ with the following properties:
\[
\begin{align*}
(\forall i \in P_{\min}) &\{\min(a_n^{-1}\{i\}) : n \in \omega\} \in R_{i, \min} \wedge \\
(\forall i \in P_{\max}) &\{\max(a_n^{-1}\{i\}) : n \in \omega\} \in R_{i, \max}.
\end{align*}
\]

Definition 2. The Tetris-finite-union closure of an element $\bar{a} \in (\text{FIN}_k)^{\omega}(\bar{R})$ is the following set
\[
\text{TFU}_k(\bar{a}) = \{T^{(j_0)}(a_{n_0}) + \cdots + T^{(j_\ell)}(a_{n_\ell}) : \\
\ell \in \omega \setminus \{0\}, n_0 < \cdots < n_\ell, j_i \in k, \exists r j_r = 0\}
\]
For $\bar{a}, \bar{b} \in (\text{FIN}_k)^{\omega}$ we let $\bar{b} \subseteq_k \bar{a}$ if $\bar{b} \subseteq \text{TFU}_k(\bar{a})$.

We sketch a proof of a common strengthening of a Theorem of Blass for $k = 1$, $PP = \{(1, \min), (1, \max)\}$ and Gowers for $k \geq 1$, $PP = \emptyset$:

Theorem 3. Let $\bar{a} \in (\text{FIN}_k)^{\omega}(\bar{R})$ and let $c$ be a colouring of $\text{TFU}_k(\bar{a})$ into finitely many colours. Then there is a $\bar{b} \subseteq_k \bar{a}$, $\bar{b} \in (\text{FIN}_k)^{\omega}(\bar{R})$ such that $\text{TFU}_k(\bar{b})$ is $c$-monochromatic.

This theorem gives rise to a new proper non-$\sigma$-centred forcing with the pure decision property.

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