wQN-space and ideal coverings of $X$

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joint work with Jaroslav Šupina

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Basic terms

**Definition (Ideal)**

The family $\mathcal{I} \subseteq \mathcal{P}(\omega)$ is called **ideal**, if it has properties:

1. $A \in \mathcal{I}, B \subseteq A \rightarrow B \in \mathcal{I},$
2. $A, B \in \mathcal{I} \rightarrow A \cup B \in \mathcal{I},$
3. $\omega \notin \mathcal{I},$
4. $(\forall n \in \omega) \{n\} \in \mathcal{I}.$

- **The Frechét ideal**, denoted as $\text{Fin}$, is a set $[\omega]^{<\aleph_0}.$
- **The Asymptotic density zero ideal**: $\mathcal{Z} = \left\{ A \subseteq \omega, \lim_{n \to \infty} \frac{|A \cap n|}{n} = 0 \right\}.$
- etc.
Definition \((\mathcal{I}\text{-convergence})\)

The sequence \(\langle f_n : n \in \omega \rangle\) of functions on \(X\) is called \(\mathcal{I}\text{-convergent}\) to a function \(f\) on \(X\) (written \(f_n \overset{\mathcal{I}}{\to} f\)), if \(\{n \in \omega : |f_n(x) - f(x)| \geq \varepsilon\} \in \mathcal{I}\) for each \(x \in X\) and for each \(\varepsilon > 0\).
Definition (\(\mathcal{I}\)-convergence)

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Definition (\(\mathcal{I}\)-quasi-normal convergence)

The sequence \(\langle f_n : n \in \omega \rangle\) is called \(\mathcal{I}\)-quasi-normal convergent to \(f\) on \(X\) if there exists a sequence of positive reals \(\langle \varepsilon_n : n \in \omega \rangle\) and \(\varepsilon_n \xrightarrow{\mathcal{I}} 0\) such that \(\{n \in \omega : |f_n(x) - f(x)| \geq \varepsilon_n\} \in \mathcal{I}\) for any \(x \in X\), denoted \(f_n \xrightarrow{\mathcal{I}QN} 0\).
Ideal Convergence

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- \(\langle \varepsilon_n : n \in \omega \rangle\) is called control sequence
- especially, if control sequence is \(\langle 2^{-n} : n \in \omega \rangle\) we are talking about strongly \(\mathcal{I}\)-quasi normal convergence of \(f_n\) to \(f\), written \(f_n \xrightarrow{s\mathcal{I}QN} f\).
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Classical convergence \(\Rightarrow\) \(\mathcal{I}\)-convergence

\(QN\)-convergence \(\Rightarrow\) \(s\mathcal{I}QN\)-convergence \(\Rightarrow\) \(\mathcal{I}QN\)-convergence
Let us consider a family $\mathcal{E}$ of functions on $X$.

$\mathcal{E}$ is closed under taking uniformly convergent series of functions from $\mathcal{E}$ and

$$\text{if } f \in \mathcal{E}, c_1, c_2 > 0 \text{ then } \min\{c_1, |c_2f|\} \in \mathcal{E}.$$
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- family of all continuous, Borel or non-negative upper semicontinuous functions $(\text{USC}_p^+(X))$ and etc.
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- family of all continuous, Borel or non-negative upper semicontinuous functions $(\text{USC}_p^+(X))$ and etc.

**Definition (\((\mathcal{I}, \mathcal{J})\text{wQN}-\text{space}\))**

A topological space $X$ is an $((\mathcal{I}, \mathcal{J})\text{wQN}$-space if for any sequence $\langle f_n : n \in \omega \rangle$ of continuous real functions $\mathcal{I}$-converging to 0 on $X$, there exists a sequence $\langle m_n : n \in \omega \rangle$ such that $f_{m_n} \overset{\mathcal{J}\text{QN}}{\rightarrow} 0$. 
Control sequences

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**Definition ((I, J)\text{wQN}-space)**

A topological space $X$ is an $\text{(I, J)wQN}\text{-space}$ if for any sequence $\langle f_n : n \in \omega \rangle$ of continuous real functions $\mathcal{I}$-converging to 0 on $X$, there exists a sequence $\langle m_n : n \in \omega \rangle$ such that $f_{m_n} \overset{\text{JQN}}{\rightarrow} 0$.

**Definition ((I, sJ)\text{wQN}(\mathcal{E})\text{-space])**

A topological space $X$ is called an $\text{(I, sJ)wQN(\mathcal{E})-space}$, if for any sequence $\langle f_n : n \in \omega \rangle$ of functions from $\mathcal{E}$ converging to zero there exists a sequence $\langle m_n : n \in \omega \rangle$ such that $f_{m_n} \overset{s\text{JQN}}{\rightarrow} 0$ with control sequence $\langle 2^{-n} : n \in \omega \rangle$.  \hfill 4 / 14
Let $\mathcal{I}, \mathcal{J}$ be ideals on $\omega$.

$$\text{wQN-space} \Rightarrow (\mathcal{I}, s\mathcal{J})\text{wQN-space} \Rightarrow (\mathcal{I}, \mathcal{J})\text{wQN-space}$$
Control sequences

Let $\mathcal{I}, \mathcal{J}$ be ideals on $\omega$.

$$wQN\text{-space } \Rightarrow (\mathcal{I}, s\mathcal{J})wQN\text{-space } \Rightarrow (\mathcal{I}, \mathcal{J})wQN\text{-space}$$

**Lemma (V.Š., J.Šupina)**

Let $\langle \varepsilon_n : n \in \omega \rangle, \langle \delta_n : n \in \omega \rangle$ be sequences of positive reals in $[0, 1]$ such that $\varepsilon_n \to 0$, $\langle \langle f_{n,m} : m \in \omega \rangle : n \in \omega \rangle$ being a sequence of sequences of functions on $X$. Then there is a sequence $\langle g_m : m \in \omega \rangle$ of functions with values in $[0, 1]$ such that the following holds.

1. If $f_{n,m} \in \mathcal{E}$ for all $n \in \omega$ then $g_m \in \mathcal{E}$, assuming $\mathcal{E}$ satisfies (1).
2. If $f_{n,m} \xrightarrow{\mathcal{I}} 0$ for each $n \in \omega$ then $g_m \xrightarrow{\mathcal{I}} 0$.
3. If $\langle f_{n,m} : m \in \omega \rangle$ are monotone sequences for each $n \in \omega$ then $\langle g_m : m \in \omega \rangle$ is a monotone sequence.
4. There is a sequence $\langle k_n : n \in \omega \rangle$ (e.g., $k_n = \max\{i, 2^{-i} \geq \varepsilon_n\}$) such that for any $x \in X$ and $m \in \omega$ we have
   $$g_m(x) < \varepsilon_n \Rightarrow |f_{k_n,m}(x)| < \frac{\delta_n}{2^{k_n}}.$$  \hspace{1cm} (2)
Control sequences

**Theorem (V.Š., J.Šupina)**

Let $X$ be a topological space. Let $E$ be a family of functions satisfying (1). Then the following are equivalent.

(a) $X$ is an $(\mathcal{I}, s\mathcal{J})wQN(E)$-space.

(b) There is a sequence $\langle \varepsilon_n : n \in \omega \rangle$ of positive reals such that $\varepsilon_n \to 0$ and for any sequence $\langle f_n : n \in \omega \rangle$ of functions from $E$ converging to zero there exists a sequence $\langle m_n : n \in \omega \rangle$ such that $f_{m_n} \overset{\mathcal{J}QN}{\longrightarrow} 0$ with control sequence $\langle \varepsilon_n : n \in \omega \rangle$.

(c) For every sequence $\langle \varepsilon_n : n \in \omega \rangle$ of positive reals such that $\varepsilon_n \to 0$ and for any sequence $\langle f_n : n \in \omega \rangle$ of functions from $E$ converging to zero there exists a sequence $\langle m_n : n \in \omega \rangle$ such that $f_{m_n} \overset{\mathcal{J}QN}{\longrightarrow} 0$ with control sequence $\langle \varepsilon_n : n \in \omega \rangle$. 

- For control sequence it is enough to ask only convergence to zero.
- Similarly for an $s\mathcal{J}wmQN(E)$-space, i.e. for $(\mathcal{F}_n, s\mathcal{J})wQN(C_p(X))$-space we consider only monotone sequences.
Theorem (V.Š., J.Šupina)

Let $X$ be a topological space. Let $\mathcal{E}$ be a family of functions satisfying (1). Then the following are equivalent.

(a) $X$ is an $(I, sJ)\, wQN(\mathcal{E})$-space.

(b) There is a sequence $\langle \varepsilon_n : n \in \omega \rangle$ of positive reals such that $\varepsilon_n \to 0$ and for any sequence $\langle f_n : n \in \omega \rangle$ of functions from $\mathcal{E}$ converging to zero there exists a sequence $\langle m_n : n \in \omega \rangle$ such that $f_{m_n} \xrightarrow{sJ\, QN} 0$ with control sequence $\langle \varepsilon_n : n \in \omega \rangle$.

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- For control sequence it is enough to ask only convergence to zero.
- Similarly for an $sJ\, wmnQN$-space, i.e. for $(\text{Fin}, sJ)\, wQN(C_p(X))$-space we consider only monotone sequences.
Ideal coverings

- \( \Omega \) denotes all \( \omega \)-covers, \( \Gamma \) denotes all \( \gamma \)-covers and \( \mathcal{O} \) denotes all open covers of \( X \).
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- \( \Omega \) denotes all \( \omega \)-covers, \( \Gamma \) denotes all \( \gamma \)-covers and \( \mathcal{O} \) denotes all open covers of \( X \).
- Let \( \mathcal{I} \) be an ideal on \( \omega \).
- A sequence \( \langle U_n : n \in \omega \rangle \) of subsets of \( X \) is called an \( \mathcal{I}-\gamma \)-cover, if for every \( n \), \( U_n \neq X \) and the set \( \{ n \in \omega ; x \notin U_n \} \in \mathcal{I} \) for every \( x \in X \), [3].
- \( \mathcal{I}-\Gamma \) denotes all \( \mathcal{I}-\gamma \)-covers of \( X \).
Ideal coverings

- $\Omega$ denotes all $\omega$-covers, $\Gamma$ denotes all $\gamma$-covers and $\mathcal{O}$ denotes all open covers of $X$.
- Let $\mathcal{I}$ be an ideal on $\omega$.
- A sequence $\langle U_n : n \in \omega \rangle$ of subsets of $X$ is called an $\mathcal{I}$-\textbf{\gamma-cover}, if for every $n$, $U_n \neq X$ and the set $\{n \in \omega; \ x \notin U_n\} \in \mathcal{I}$ for every $x \in X$, [3].
- $\mathcal{I}$-$\Gamma$ denotes all $\mathcal{I}$-$\gamma$-covers of $X$.
- Let $\mathcal{A}$ and $\mathcal{B}$ be families of sets.
- $S_1(\mathcal{A}, \mathcal{B})$: for a sequence $\langle U_n : n \in \omega \rangle$ of elements of $\mathcal{A}$ we can select a set $U_n \in U_n$ for each $n \in \omega$ such that $\langle U_n : n \in \omega \rangle$ is a sequence of $\mathcal{B}$
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For topological space $X$

$$
\begin{array}{ccc}
S_1(\Omega, \Gamma) & \downarrow & \\
\downarrow & & \\
S_1(\mathcal{I}\Gamma, \Gamma) & \longrightarrow & S_1(\mathcal{I}\Gamma, \mathcal{J}\Gamma) \\
\downarrow & & \downarrow \\
S_1(\Gamma, \Gamma) & \longrightarrow & S_1(\Gamma, \mathcal{J}\Gamma) & \longrightarrow & S_1(\Gamma, \Omega)
\end{array}
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**Lemma (J. Šupina)**

For any countable $\omega$-cover $\mathcal{U}$ and its bijective enumeration $\langle U_n : n \in \omega \rangle$ there is an ideal $\mathcal{I}$ such that $\langle U_n : n \in \omega \rangle$ is an $\mathcal{I}$-$\gamma$-cover.
Similarly to M. Scheepers [7] we define

\[ \mathcal{I}-\Gamma_x(X) = \{ A \in [X \setminus \{x\}]^\omega; \ A \text{ is } \mathcal{I}\text{-convergent to } x \}. \]
Similarly to M. Scheepers [7] we define

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Consider the \( C_p(X) \) - a set of continuous functions from \( X \) to \( \mathbb{R} \) endowed with the Tychonoff product topology.

0 denotes the function on \( X \) which is equal to zero everywhere.

\[ \mathcal{I}-\Gamma_0(C_p(X)) = \mathcal{I}-\Gamma_0. \]
Similarly to M. Scheepers [7] we define

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- Consider the \( C_p(X) \) - a set of continuous functions from \( X \) to \( \mathbb{R} \) endowed with the Tychonoff product topology.
- \( 0 \) denotes the function on \( X \) which is equal to zero everywhere.
- \( \mathcal{I}^{-\Gamma}_0( C_p(X) ) = \mathcal{I}^{-\Gamma}_0 \).

**Definition**

The \( C_p(X) \) has **the property** \( S_1(\mathcal{I}^{-\Gamma}_0, \mathcal{J}^{-\Gamma}_0) \) if:

for any sequence \( \langle \langle f_{n,m}: \; m \in \omega \rangle: \; n \in \omega \rangle \) of sequences of continuous real functions such that \( f_{n,m} \xrightarrow{\mathcal{I}} 0 \) for each \( n \), there exists a sequence \( \langle m_n: \; n \in \omega \rangle \) such that \( f_{n,m_n} \xrightarrow{\mathcal{J}} 0 \).
Ideal coverings

Definition

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for any sequence $\langle \langle f_{n,m} : m \in \omega \rangle : n \in \omega \rangle$ of sequences of continuous real functions such that $f_{n,m} \xrightarrow{\mathcal{I}} 0$ for each $n$, there exists a sequence $\langle m_n : n \in \omega \rangle$ such that $f_{n,m_n} \xrightarrow{\mathcal{J}} 0$.

\[
\begin{array}{cccccc}
\text{Fréchet} & \longrightarrow & S_1(\mathcal{I} - \Gamma_0, \Gamma_0) & \longrightarrow & S_1(\mathcal{I} - \Gamma_0, \mathcal{J} - \Gamma_0) \\
& & \downarrow & & \downarrow \\
& & S_1(\Gamma_0, \Gamma_0) & \longrightarrow & S_1(\Gamma_0, \mathcal{J} - \Gamma_0) & \longrightarrow & S_1(\Gamma_0, \Omega_0) & \longrightarrow & \text{Ind}_{\mathbb{Z}}(X) = 0
\end{array}
\]
Definition

We say that a topological space $X$ has $\mathcal{J}$-Hurewicz property if for each sequence $\langle U_n : n \in \omega \rangle$ of open covers of $X$ there are finite $V_n \subseteq U_n$, $n \in \omega$ such that for each $x \in X$, $\{n \in \omega, x \notin \bigcup V_n\} \in \mathcal{J}$. 
Ideal coverings

**Definition**

We say that a topological space $X$ has **$J$-Hurewicz property** if for each sequence $\langle U_n : n \in \omega \rangle$ of open covers of $X$ there are finite $V_n \subset U_n$, $n \in \omega$ such that for each $x \in X$, $\{ n \in \omega, \ x \notin \bigcup V_n \} \in J$.

- $J$-Hurewicz property was introduced by P. Das [3].
- P. Szewczak and B. Tsaban [9] (they consider an $S$-Menger property) showed that

\[
\text{Hurewicz} \rightarrow J\text{-Hurewicz} \rightarrow \text{Menger}
\]
Definition

We say that a topological space $X$ has **$\mathcal{J}$-Hurewicz property** if for each sequence $\langle \mathcal{U}_n : n \in \omega \rangle$ of open covers of $X$ there are finite $\mathcal{V}_n \subseteq \mathcal{U}_n$, $n \in \omega$ such that for each $x \in X$, $\{ n \in \omega, x \notin \bigcup \mathcal{V}_n \} \in \mathcal{J}$.

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$$\text{Hurewicz} \rightarrow \mathcal{J}\text{-Hurewicz} \rightarrow \text{Menger}$$

Theorem (Bukovský–Das–Šupina)

*If* $X$ *is a normal topological space then the following are equivalent. Moreover, the equivalence (a) $\equiv$ (b) holds for arbitrary topological space $X$.*

(a) $X$ is an $(\mathcal{I}, s\mathcal{J})w\text{QN}$-space.

(b) $C_p(X)$ has the property $S_1(\mathcal{I}\text{-}\Gamma_0, \mathcal{J}\text{-}\Gamma_0)$.

(c) $X$ is an $S_1(\mathcal{I}\text{-}\Gamma^sh, \mathcal{J}\text{-}\Gamma)$-space.
Ideal coverings

Theorem (V.Š., J.Šupina)

If $X$ is a perfectly normal topological space then the following are equivalent. Moreover, if $X$ is arbitrary topological space then $(a) \equiv (b)$.

(a) $X$ is an $s \mathcal{J}$wmQN-space.

(b) $C_p(X)$ has the property $S_1(\Gamma_0^m, \mathcal{J} - \Gamma_0)$.

(c) $X$ possesses a $\mathcal{J}$-Hurewicz property.
Ideal coverings

**Theorem (V.Š., J.Šupina)**

If $X$ is a perfectly normal topological space then the following are equivalent. Moreover, if $X$ is arbitrary topological space then (a) $\equiv$ (b).

(a) $X$ is an $sJ\text{wmQN}$-space.
(b) $C_p(X)$ has the property $S_1(\Gamma_0^m, J-\Gamma_0)$.
(c) $X$ possesses a $J$-Hurewicz property.

**Theorem (V.Š., J.Šupina)**

Let $\mathcal{I}, \mathcal{J}$ be ideals on $\omega$. Then the following statements are equivalent.

(a) $X$ is an $S_1(\mathcal{I}-\Gamma, \mathcal{J}-\Gamma)$-space.
(b) $\text{USC}_p^+(X)$ has the property $S_1(\mathcal{I}-\Gamma_0, \mathcal{J}-\Gamma_0)$.
(c) $X$ is an $(\mathcal{I}, s\mathcal{J})\text{wQN}(\text{USC}_p^+(X))$-space.
## Ideal coverings

<table>
<thead>
<tr>
<th>$X$ is</th>
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<tbody>
<tr>
<td>$(\mathcal{I}, s\mathcal{J})_{wQN}$-space</td>
<td>$S_1(\mathcal{I}-\Gamma, \mathcal{J}-\Gamma)$</td>
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<tr>
<td>$s\mathcal{J}_{wmQN}$-space</td>
<td>$\mathcal{J}$-Hurewicz</td>
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<td>$(\mathcal{I}, s\mathcal{J})_{wQN(USC_p(X))}$-space</td>
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Thank you for your attention

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