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Faculty of Science

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wQN-space and ideal coverings of X

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Definition (Ideal)

The family $\mathcal{I} \subseteq \mathcal{P}(\omega)$ is called **ideal**, if it has properties:

(I1) $A \in \mathcal{I},\ B \subseteq A \to B \in \mathcal{I}$,

(I2) $A, B \in \mathcal{I} \to A \cup B \in \mathcal{I}$,

(I3) $\omega \notin \mathcal{I}$,

(I4) $(\forall n \in \omega)\ \{n\} \in \mathcal{I}$.

- **The Frechét ideal**, denoted as Fin, is a set $[\omega]^{<\aleph_0}$.
- **The Asymptotic density zero ideal**: $\mathcal{Z} = \left\{A \subseteq \omega,\ \lim_{n\to\infty} \frac{|A\cap n|}{n} = 0\right\}$.
- etc.

Ideal Convergence

Definition ($\mathcal{I}$-convergence)

The sequence $\langle f_n : n \in \omega \rangle$ of functions on X is called $\mathcal{I}$-**convergent** to a function f on X (written $f_n \xrightarrow{\mathcal{I}} f$), if $\{n \in \omega : |f_n(x) - f(x)| \geq \varepsilon\} \in \mathcal{I}$ for each $x \in X$ and for each $\varepsilon > 0$.

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Definition ($\mathcal{I}$-quasi-normal convergence)

The sequence $\langle f_n : n \in \omega \rangle$ is called $\mathcal{I}$-**quasi-normal convergent** to f on X if there exists a sequence of positive reals $\langle \varepsilon_n : n \in \omega \rangle$ and $\varepsilon_n \xrightarrow{\mathcal{I}} 0$ such that $\{n \in \omega : |f_n(x) - f(x)| \geq \varepsilon_n\} \in \mathcal{I}$ for any $x \in X$, denoted $f_n \xrightarrow{\mathcal{I}\mathrm{QN}} 0$.

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- $\langle \varepsilon_n : n \in \omega \rangle$ is called control sequence
- especially, if control sequence is $\langle 2^{-n} : n \in \omega \rangle$ we are talking about **strongly** $\mathcal{I}$-**quasi normal convergence** of f_n to f, written $f_n \xrightarrow{\mathrm{s}\mathcal{I}\mathrm{QN}} f$.

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classical convergence $\Rightarrow$ $\mathcal{I}$-convergence

QN-convergence $\Rightarrow$ s$\mathcal{I}\mathrm{QN}$-convergence $\Rightarrow$ $\mathcal{I}\mathrm{QN}$-convergence

Control sequences

Let us consider a family $\mathcal{E}$ of functions on X.

$$\begin{gathered}\mathcal{E} \text{ is closed under taking uniformly convergent series of functions from } \mathcal{E} \\ \text{and} \\ \text{if } f \in \mathcal{E}, c_1, c_2 > 0 \text{ then } \min\{c_1, |c_2 f|\} \in \mathcal{E}.\end{gathered} \tag{1}$$

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Definition ($(\mathcal{I}, \mathcal{J})\mathrm{wQN}$-space)

A topological space X is an $(\mathcal{I}, \mathcal{J})\mathrm{wQN}$-**space** if for any sequence $\langle f_n : n \in \omega \rangle$ of continuous real functions $\mathcal{I}$-converging to 0 on X, there exists a sequence $\langle m_n : n \in \omega \rangle$ such that $f_{m_n} \xrightarrow{\mathcal{J}\mathrm{QN}} 0$.

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Definition ($(\mathcal{I},\mathrm{s}\mathcal{J})\mathrm{wQN}(\mathcal{E})$-space)

A topological space X is called an $(\mathcal{I},\mathbf{s}\mathcal{J})\mathrm{wQN}(\mathcal{E})$-**space**, if for any sequence $\langle f_n : n \in \omega\rangle$ of functions from $\mathcal{E}$ converging to zero there exists a sequence $\langle m_n : n \in \omega\rangle$ such that $f_{m_n} \xrightarrow{\mathrm{s}\mathcal{J}\mathrm{QN}} 0$ with control sequence $\langle 2^{-n} : n \in \omega\rangle$.

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Let $\mathcal{I}, \mathcal{J}$ be ideals on ω.

$$\text{wQN-space} \Rightarrow (\mathcal{I}, \mathsf{s}\mathcal{J})\text{wQN-space} \Rightarrow (\mathcal{I}, \mathcal{J})\text{wQN-space}$$

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Lemma (V.Š.,J.Šupina)

Let $\langle \varepsilon_n : \ n \in \omega \rangle$, $\langle \delta_n : \ n \in \omega \rangle$ be sequences of positive reals in $[0,1]$ such that $\varepsilon_n \to 0$, $\langle \langle f_{n,m} : \ m \in \omega \rangle : \ n \in \omega \rangle$ being a sequence of sequences of functions on X. Then there is a sequence $\langle g_m : \ m \in \omega \rangle$ of functions with values in $[0,1]$ such that the following holds.

(1) *If $f_{n,m} \in \mathcal{E}$ for all $n \in \omega$ then $g_m \in \mathcal{E}$, assuming $\mathcal{E}$ satisfies* (1).

(2) *If $f_{n,m} \xrightarrow{\mathcal{I}} 0$ for each $n \in \omega$ then $g_m \xrightarrow{\mathcal{I}} 0$.*

(3) *If $\langle f_{n,m} : \ m \in \omega \rangle$ are monotone sequences for each $n \in \omega$ then $\langle g_m : \ m \in \omega \rangle$ is a monotone sequence.*

(4) *There is a sequence $\langle k_n : \ n \in \omega \rangle$ (e.g., $k_n = \max\{i, \ 2^{-i} \geq \varepsilon_n\}$) such that for any $x \in X$ and $m \in \omega$ we have*

$$g_m(x) < \varepsilon_n \to \ |f_{k_n,m}(x)| < \frac{\delta_n}{2^{k_n}}. \tag{2}$$

Theorem (V.Š.,J.Šupina)

Let X be a topological space. Let $\mathcal{E}$ be a family of functions satisfying (1). Then the following are equivalent.

(a) *X is an $(\mathcal{I}, s\mathcal{J})\mathrm{wQN}(\mathcal{E})$-space.*

(b) *There is a sequence $\langle \varepsilon_n : n \in \omega \rangle$ of positive reals such that $\varepsilon_n \to 0$ and for any sequence $\langle f_n : n \in \omega \rangle$ of functions from $\mathcal{E}$ converging to zero there exists a sequence $\langle m_n : n \in \omega \rangle$ such that $f_{m_n} \xrightarrow{\mathcal{J}QN} 0$ with control sequence $\langle \varepsilon_n : n \in \omega \rangle$.*

(c) *For every sequence $\langle \varepsilon_n : n \in \omega \rangle$ of positive reals such that $\varepsilon_n \to 0$ and for any sequence $\langle f_n : n \in \omega \rangle$ of functions from $\mathcal{E}$ converging to zero there exists a sequence $\langle m_n : n \in \omega \rangle$ such that $f_{m_n} \xrightarrow{\mathcal{J}QN} 0$ with control sequence $\langle \varepsilon_n : n \in \omega \rangle$.*

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- For control sequence it is enough to ask only convergence to zero.
- Similarly for an $\mathrm{s}\mathcal{J}\mathrm{wmQN}$-space, i.e. for $(\mathrm{Fin}, \mathrm{s}\mathcal{J})\mathrm{wQN}(C_p(X))$-space we consider only monotone sequences.

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- Ω denotes all ω-covers, Γ denotes all γ-covers and $\mathcal{O}$ denotes all open covers of X.

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- Let $\mathcal{I}$ be an ideal on ω.
- A sequence $\langle U_n : \ n \in \omega \rangle$ of subsets of X is called an $\mathcal{I}$-γ-**cover**, if for every $n, \ U_n \neq X$ and the set $\{n \in \omega; \ x \notin U_n\} \in \mathcal{I}$ for every $x \in X$, [3].
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- Let $\mathcal{A}$ and $\mathcal{B}$ be families of sets.
- $S_1(\mathcal{A}, \mathcal{B})$: for a sequence $\langle \mathcal{U}_n : \ n \in \omega \rangle$ of elements of $\mathcal{A}$ we can select a set $U_n \in \mathcal{U}_n$ for each $n \in \omega$ such that $\langle U_n : \ n \in \omega \rangle$ is a sequence of $\mathcal{B}$

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For topological space X

$$\begin{array}{ccccc}
\mathrm{S}_1(\Omega, \Gamma) & & & & \\
\downarrow & & & & \\
\mathrm{S}_1(\mathcal{I}\text{-}\Gamma, \Gamma) & \longrightarrow & \mathrm{S}_1(\mathcal{I}\text{-}\Gamma, \mathcal{J}\text{-}\Gamma) & & \\
\downarrow & & \downarrow & & \\
\mathrm{S}_1(\Gamma, \Gamma) & \longrightarrow & \mathrm{S}_1(\Gamma, \mathcal{J}\text{-}\Gamma) & \longrightarrow & \mathrm{S}_1(\Gamma, \Omega)
\end{array}$$

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Lemma (J.Šupina)

For any countable ω-cover $\mathcal{U}$ and its bijective enumeration $\langle U_n : \ n \in \omega \rangle$ there is an ideal $\mathcal{I}$ such that $\langle U_n : \ n \in \omega \rangle$ is an $\mathcal{I}$-γ-cover.

Ideal coverings

- Similarly to M. Scheepers [7] we define

$$\mathcal{I}\text{-}\Gamma_x(X) = \{A \in [X \setminus \{x\}]^\omega;\ A \text{ is } \mathcal{I}\text{-convergent to } x\}.$$

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- Consider the $C_p(X)$ - a set of continuous functions from X to $\mathbb{R}$ endowed with the Tychonoff product topology.
- **0** denotes the function on X which is equal to zero everywhere.
- $\mathcal{I}\text{-}\Gamma_{\mathbf{0}}(C_p(X)) = \mathcal{I}\text{-}\Gamma_{\mathbf{0}}$.

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Definition

The $C_p(X)$ has **the property** $\mathrm{S}_1(\mathcal{I}\text{-}\Gamma_{\mathbf{0}}, \mathcal{J}\text{-}\Gamma_{\mathbf{0}})$ if:

for any sequence $\langle\langle f_{n,m}:\ m \in \omega\rangle :\ n \in \omega\rangle$ of sequences of continuous real functions such that $f_{n,m} \xrightarrow{\mathcal{I}} 0$ for each n, there exists a sequence $\langle m_n :\ n \in \omega\rangle$ such that $f_{n,m_n} \xrightarrow{\mathcal{J}} 0$.

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The $C_p(X)$ has **the property** $\mathrm{S}_1(\mathcal{I}\text{-}\Gamma_{\mathbf{o}}, \mathcal{J}\text{-}\Gamma_{\mathbf{o}})$ if:

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$$
\begin{array}{ccccccc}
\text{Fréchet} & \longrightarrow & \mathrm{S}_1(\mathcal{I}\text{-}\Gamma_{\mathbf{o}}, \Gamma_{\mathbf{o}}) & \rightarrow & \mathrm{S}_1(\mathcal{I}\text{-}\Gamma_{\mathbf{o}}, \mathcal{J}\text{-}\Gamma_{\mathbf{o}}) & & \\
 & & \downarrow & & \downarrow & & \\
 & & \mathrm{S}_1(\Gamma_0, \Gamma_0) & \longrightarrow & \mathrm{S}_1(\Gamma_{\mathbf{o}}, \mathcal{J}\text{-}\Gamma_{\mathbf{o}}) & \longrightarrow & \mathrm{S}_1(\Gamma_{\mathbf{o}}, \Omega_{\mathbf{o}}) \longrightarrow \mathsf{Ind}_{\mathbb{Z}}(X) = 0
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Definition

We say that a topological space X has $\mathcal{J}$-**Hurewicz property** if for each sequence $\langle \mathcal{U}_n : \ n \in \omega \rangle$ of open covers of X there are finite $\mathcal{V}_n \subset \mathcal{U}_n$, $n \in \omega$ such that for each $x \in X$, $\{n \in \omega, \ x \notin \bigcup \mathcal{V}_n\} \in \mathcal{J}$.

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- $\mathcal{J}$-Hurewicz property was introduced by P. Das [3].
- P. Szewczak and B. Tsaban [9] (they consider an $\mathcal{S}$-Menger property) showed that

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Theorem (Bukovský–Das–Šupina)

If X is a normal topological space then the following are equivalent. Moreover, the equivalence $(\mathrm{a}) \equiv (\mathrm{b})$ holds for arbitrary topological space X.

(a) *X is an $(\mathcal{I}, s\mathcal{J})\mathrm{wQN}$-space.*

(b) *$C_p(X)$ has the property $\mathrm{S}_1(\mathcal{I}\text{-}\Gamma_{\mathbf{0}}, \mathcal{J}\text{-}\Gamma_{\mathbf{0}})$.*

(c) *X is an $S_1(\mathcal{I}\text{-}\Gamma^{sh}, \mathcal{J}\text{-}\Gamma)$-space.*

Theorem (V.Š.,J.Šupina)

If X is a perfectly normal topological space then the following are equivalent. Moreover, if X is arbitrary topological space then $\mathrm{(a)} \equiv \mathrm{(b)}$.

(a) *X is an $\mathrm{s}\mathcal{J}\mathrm{wmQN}$-space.*

(b) *$C_p(X)$ has the property $\mathrm{S}_1(\Gamma_{\mathbf{0}}^m, \mathcal{J}\text{-}\Gamma_{\mathbf{0}})$.*

(c) *X possesses a $\mathcal{J}$-Hurewicz property.*

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If X is a perfectly normal topological space then the following are equivalent. Moreover, if X is arbitrary topological space then $(\mathrm{a}) \equiv (\mathrm{b})$.

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(c) X *possesses a* $\mathcal{J}$*-Hurewicz property.*

Theorem (V.Š.,J.Šupina)

Let $\mathcal{I}, \mathcal{J}$ *be ideals on* ω*. Then the following statements are equivalent.*

(a) X *is an* $\mathrm{S}_1(\mathcal{I}\text{-}\Gamma, \mathcal{J}\text{-}\Gamma)$*-space.*

(b) $\mathrm{USC}_p^+(X)$ *has the property* $\mathrm{S}_1(\mathcal{I}\text{-}\Gamma_{\mathbf{0}}, \mathcal{J}\text{-}\Gamma_{\mathbf{0}})$.

(c) X *is an* $(\mathcal{I}, s\mathcal{J})\mathrm{wQN}(\mathrm{USC}_p^+(X))$*-space.*

Ideal coverings

X is	X is	
$(\mathcal{I}, \mathsf{s}\mathcal{J})\mathrm{wQN}$-space	$\mathrm{S}_1(\mathcal{I}\text{-}\Gamma, \mathcal{J}\text{-}\Gamma)$	$C_p(X)$ has $\mathrm{S}_1(\mathcal{I}\text{-}\Gamma_{\mathbf{0}}, \mathcal{J}\text{-}\Gamma_{\mathbf{0}})$
$\mathrm{s}\mathcal{J}\mathrm{wmQN}$-space	$\mathcal{J}$-Hurewicz	$C_p(X)$ has $\mathrm{S}_1(\Gamma^m_{\mathbf{0}}, \mathcal{J}\text{-}\Gamma_{\mathbf{0}})$
$(\mathcal{I}, \mathsf{s}\mathcal{J})\mathrm{wQN}(\mathsf{USC}_p(X))$-space	$\mathrm{S}_1(\mathcal{I}\text{-}\Gamma, \mathcal{J}\text{-}\Gamma)$	$\mathsf{USC}_p(X)$ has $\mathrm{S}_1(\mathcal{I}\text{-}\Gamma_{\mathbf{0}}, \mathcal{J}\text{-}\Gamma_{\mathbf{0}})$

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Thank you for your attention

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