A tree $T$ on $\omega$ is called

- Sacks tree or perfect tree, denoted by $T \in S$, if for each node $s \in T$ there is $t \in T$ such that $s \subseteq t$ and $|\text{succ}(t)| \geq 2$;
- Miller tree or superperfect tree, denoted by $T \in M$, if for each node $s \in T$ exists $t \in T$ such that $s \subseteq t$ and $|\text{succ}(t)| = \aleph_0$;
- Laver tree, denoted by $T \in L$, if for each node $t \supseteq \text{stem}(T)$ we have $|\text{succ}(t)| = \aleph_0$;
- complete Laver tree, denoted by $T \in CL$, if $T$ is Laver and $\text{stem}(T) = \emptyset$;

Let $\mathbb{T}$ be a family of trees. Then we define a tree ideal $t_0$ as follows:

**Definition 1.** Let $X \subseteq \omega^\omega$. Then

$$X \in t_0 \iff (\forall T \in \mathbb{T})(\exists T' \subseteq T, T' \in \mathbb{T})(T' \cap X = \emptyset).$$

For example $s_0$ is the classic Marczewski ideal.

Let us recall a notion of $\mathcal{I}$-Luzin sets.

**Definition 2.** Let $X$ be a Polish space and $\mathcal{I}$ be an ideal. Then we call a set $L \subseteq X$ an $\mathcal{I}$-Luzin set if $|L \cap A| < |L|$ for all $A \in \mathcal{I}$.

For classic ideals of Lebesgue measure zero sets $\mathcal{N}$ and meager sets $\mathcal{M}$ we will call $\mathcal{M}$-Luzin sets generalized Luzin sets and $\mathcal{N}$-Luzin sets generalized Sierpiński sets. We will work on the real line $\mathbb{R}$ with addition. Since $\mathbb{R}$ is $\sigma$-compact, it does not contain even bodies of Miller trees. We will tweak the definition a bit by saying that $A \subseteq \mathbb{R}$ belongs to $t_0$ if $h^{-1}[A]$ belongs to $t_0$ in $\omega^\omega$, where $h$ is a homeomorphism between $\omega^\omega$ and a subspace of irrational numbers (see [1] for a similar modification in the case of $2^\omega$). Using a subtle kind of fusion for Miller and Laver trees we will prove that

**Lemma 1.** There exists a dense $G_\delta$ set $G$ such that for each Miller (resp. Laver or complete Laver) tree $T$ there exists a Miller (resp. Laver or complete Laver) subtree $T' \subseteq T$ such that $G + [T'] \in \mathcal{N}$.

We will use this result to obtain the following theorem that extends the result achieved in [3].

**Theorem 1.** Let $\mathfrak{c}$ be a regular cardinal and $t_0 \in \{s_0, m_0, l_0, cl_0\}$. Then for every generalized Luzin set $L$ and generalized Sierpiński set $S$ we have $L + S \in t_0$.

Results are available in [2].
References

