Haar-small sets

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Łódź University of Technology

Winter School in Abstract Analysis, Hejnice 2018

joint work with Eliza Jabłońska, Taras Banakh and Szymon Głąb (in progress)

Motivation - Steinhaus part

Start with apologizing!

Motivation - Haar-small part

Same as in June!

Motivation - Haar-small part

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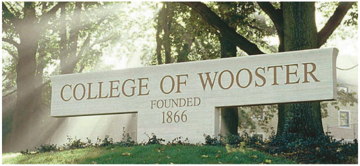

Black Squirrel Symposium

XLI. SUMMER SYMPOSIUM IN REAL ANALYSIS

June 18–24, 2017 The College of Wooster Wooster Ohio USA

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Bruce Hanson
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Sobolev Institute, Novosibirsk

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University of Illinois Urbana-Champaign

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Theory of Real Functions

Geometric Measure Theory

Descriptive Set Theory

Banach Spaces

Dynamical Systems

Practical Information

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Budapest

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So the key is that

as you can see, I am not an expert in being small, but I would strongly appreciate getting some knowledge in area :-)

Haar-smallness

We look for measure-like notion of smallness for X.

Hence in locally compact Polish groups we obtain natural notion of null sets.

If considered group is not locally compact there is no distinguished measure on it, however analogous notion (so-called Haar null sets) were introduced by Christensen in 1972. His notion was rediscovered by Hunt, Sauer and Yorke in 1992 and since that time it was deeply investigated.

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Haar-null sets

We say that set $A \subset X$ is *Haar-null*, or $A \in \mathcal{HN}(X)$, if there exists Borel hull $B \supset A$ and a Borel probalistic measure μ on X such that for any $x \in X$ we have $\mu(B + x) = 0$. We say that μ witnesses the fact that A is Haar-null. In original Christensen's paper hull B was supposed only to be universally measureable.

Theorem (Christensen)

Haar-null sets forms a proper σ-ideal. If X is locally compact, they coincide with Haar-measure null sets.

Observation

For any $A \in \mathcal{HN}$:

- measure which witnesses this fact is continuous,
- A has empty interior,
- there exist compactly (or even Cantorly) supported measure which witnesses this fact.

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Theorem

For a Borel subset A in a Polish group X the following conditions are equivalent:

- A is Haar-null in X,
- there exists an injective continuous map $f : 2^\omega \to X$ such that $f^{-1}(A + x) \in \mathcal{N}$ for all $x \in X$,
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Few words about proof.

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Haar-meager sets

Definition (Darji 2014)

A Borel subset B of a Polish group X is called

- *Haar-meager* if there exists a continuous function $f : 2^\omega \to X$ such that $f^{-1}(B+x)$ is meager in 2^ω for each $x \in X$;
- *injectively Haar-meager* if there exists an injective continuous function $f : 2^\omega \to X$ such that $f^{-1}(B+x)$ is meager in 2^ω for each $x \in X$;
- *strongly Haar-meager* if there exists a non-empty compact subset $K \subset X$ such that the set $K \cap (B+x)$ is meager in K for each $x \in X$.

Theorem (Darji)

For any Polish group X the family $\mathcal{HM}$ is a σ-ideal, contained in $\mathcal{M}$.

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The following conditions are equivalent:

- a Polish group X is locally compact,
- $\mathcal{HM} = \mathcal{M}$,
- $\overline{\mathcal{HM}} = \overline{\mathcal{M}}$.

Problem

Is $\mathcal{EHM}$ a σ-ideal? Is $\mathcal{SHM}$ a σ-ideal? Are there any equalities?

Remark

For any totally disconnected Polish group X we get $\mathcal{EHM} = \mathcal{SHM}$.

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A topological group X is called *hull-compact* if each compact subset of X is contained in a compact subgroup of X.

Theorem

Each hull-compact Polish group has $\mathcal{HM} = \mathcal{SHM}$.

Example

The Tychonoff product $X = \prod_{n\in\omega} X_n$ of infinite locally finite discrete groups X_n is Polish, hull-compact, but not locally compact. For this group we have $\mathcal{EHM} = \mathcal{SHM} = \mathcal{HM} \neq \mathcal{M}$.

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Theorem

For any non-empty analytic subspace $\mathcal{K} \subset \mathcal{K}(\mathbb{R}^\omega)$ there exists a closed set $F \subset \mathbb{R}^\omega$ such that

1. for any $K \in \mathcal{K}$ there exists $x \in \mathbb{R}^\omega$ such that $K + x \subset F$;
2. for any $x \in \mathbb{R}^\omega$ the intersection $F \cap (x + [0,1]^\omega)$ is contained in $K + d$ for some $K \in \mathcal{K}$ and $d \in \mathbb{R}^\omega$.

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Polish group $\mathbb{R}^\omega$ contains a closed subset $F \in \mathcal{SHM} \setminus \mathcal{EHM}$.

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Definition

Let $\mathcal{I}$ be an semi-ideal on 2^ω. We say that $A \subset X$ is $\mathcal{HI}$ if there exists Borel $B \supset A$ and continuous $f : 2^\omega \to X$ that $f^{-1}(B + x) \in \mathcal{I}$ for every $x \in X$.

Theorem

Let $\mathcal{I}$ be a proper ideal on 2^ω. Any closed Haar-$\mathcal{I}$ set A in a Polish group X is (strongly) Haar-meager, which yields the inclusions $\overline{\mathcal{HI}} \subset \overline{\mathcal{HM}}$ and $\sigma\overline{\mathcal{HI}} \subset \sigma\overline{\mathcal{HM}}$.

Theorem

For any proper σ-ideal $\mathcal{I}$ on a 2^ω there exists such a subideal $\mathcal{J} \subset \mathcal{I}$ that each set $J \in \mathcal{J}$ has empty interior and $\mathcal{HI} = \mathcal{HJ}$.

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Theorem

Let $h: X \to Y$ be a continuous surjective homomorphism between Polish groups. For any (injectively) Haar-$\mathcal{I}$ set $A \subset Y$, the preimage $h^{-1}(A)$ is an (injectively) Haar-$\mathcal{I}$ set in X. Just one direction

Proposition

For each ideal $\mathcal{I}$, $\varepsilon > 0$ and $A \in \mathcal{HI}$ there exists $f \colon 2^\omega \to B(\theta, \varepsilon)$ witnessing that fact.

Problem

When $\mathcal{HI}$ is an ideal?

Example, A. Kwela

Not when $\mathcal{I} = Fin$, $X = \mathbb{R}$. Thus also for any $X = \mathbb{R} \times H$.

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In general case

Definition

Each family $\mathcal{I}$ of subsets of the space 2^ω induces the families

$$\mathcal{I}_i^n = \{A \subset (2^\omega)^n : \forall a \in (2^\omega)^{n\setminus\{i\}} \ \ e_a^{-1}(A) \in \mathcal{I}\}.$$

We say that $\mathcal{I}$ is *n-Fubini* for $n \in \mathbb{N} \cup \{\omega\}$ if there exists a continuous map $h : 2^\omega \to (2^\omega)^n$ such that for any $i \in n$ and any Borel set $B \in \mathcal{I}_i^n$ the preimage $h^{-1}(B)$ belongs to the family $\mathcal{I}$.

n-Fubini are equivalent for all $n \in \{1, 2, \ldots, \omega\}$.

Theorem

For any Fubini (σ-)ideal $\mathcal{I}$ on 2^ω, $\mathcal{HI}$ is also (σ-)ideal.

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For any Fubini (σ-)ideal $\mathcal{I}$ on 2^ω, $\mathcal{H}\mathcal{I}$ is also (σ-)ideal.

Towards generics! (just a bit)

$$W_{\mathcal{I}}(A) = \{f \in C((2^\omega), X) : \forall x \in X \;\; f^{-1}(A + x) \in \mathcal{I}\}$$

Theorem

For ideal $\mathcal{I}$ on 2^ω and any $A \subset X$ exactly one of following holds:

1. $W_{\mathcal{I}}(A)$ is empty;
2. $W_{\mathcal{I}}(A)$ meager and dense in $C(K, X)$;
3. $W_{\mathcal{I}}(A)$ is a dense Baire subspace of $C(K, X)$.

Theorem

A σ-compact subset of a Polish group is generically Haar-null(meager) iff it is Haar-null(meager).

Definition

We say that $\mathcal{I}$ is generically Haar-$\mathcal{I}$ if $W_{\mathcal{I}}(A)$ is comeager in $C(2^\omega, X)$.

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If $\mathcal{I}$ is (σ)-ideal, then so is $\mathcal{GHI}$.

Theorem

A subset A of a non-discrete Polish group X is

1. $\mathcal{GHN}$ iff $T(A) := \{\mu \in P(X) : \forall x \in X \;\; \mu(A + x) = 0\}$ is comeager in the space $P(X)$;
2. $\mathcal{GHM}$ iff $K_{\mathcal{M}}(A) := \{K \in \mathcal{K}(X) : \forall x \in X \;\; K \cap (A + x) \in \mathcal{M}_K\}$ is comeager in $\mathcal{K}(X)$.

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Theorem

A subset A of a non-discrete Polish group X is

1. $\mathcal{GHN}$ iff $T(A) := \{\mu \in P(X) : \forall x \in X \ \ \mu(A + x) = 0\}$ is comeager in the space $P(X)$;
2. $\mathcal{GHM}$ iff $K_{\mathcal{M}}(A) := \{K \in \mathcal{K}(X) : \forall x \in X \ \ K \cap (A + x) \in \mathcal{M}_K\}$ is comeager in $\mathcal{K}(X)$.

Open problem - with prize!

For $\mathcal{F} \subset \mathcal{P}(X)$ and $A \subset X$ we say that A is $\mathcal{F}$-Haar-meager ($A \in \mathcal{HM}(\mathcal{F})$) if there exists such $B \supset A,\ B \in \mathcal{F}$ and $f \in C(2^\omega, X)$ that for each $x \in X$ we have $f^{-1}(B + x) \in \mathcal{M}$.

Consider families:

$$\mathcal{F}_1 := \{B \subset X : \forall_{(T\text{ - top.})} \forall_{f \in C(T,X)}\ f^{-1}(B) \text{ is Baire set}\}$$

$$\mathcal{F}_2 := \{B \subset X : \forall_{(T\text{ - Polish})} \forall_{f \in C(T,X)}\ f^{-1}(B) \text{ is Baire set}\}$$

Problem (Old, solved by M.Goldstern)

Is $\mathcal{HM}(\mathcal{F}_1) = \mathcal{HM}(\mathcal{F}_1)$?

Prize(granted)

One bottle of Polish mead/miód pitny/medovina/Honigwein.

Problem (correct)

Is $\mathcal{HM}(\mathcal{F}_1) = \mathcal{HM}(\mathcal{F}_2)$?

Prize

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Děkuji za pozornost! Köszönöm a figyelmet!
Thank you for your attention! Dziękuję za uwagę!
Obrigado pela atenção!
Ďakujem za vašu pozornosť! Дякую за увагу!
Merci de votre attention ! תודה לך על תשומת הלב
Gratias pro vobis animus attentus!
Danke für Ihre Aufmerksamkeit!
Спасибо за внимание!
Hvala za vašo pozornost! با تشکر از توجه شما
Σας ευχαριστώ για την προσοχή σας!
ध्यान देने के एलधिन्यवाद!
ขอขอบคุณสำหรับความสนใจของคุณ! 感谢您的关注！