GROTHENDIECK $C(K)$-SPACES OF SMALL DENSITY

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A common issue in functional analysis is to study when the convergence of sequences in a topology τ_1 on a given Banach space X implies the convergence in a topology τ_2 where $\tau_1 \subseteq \tau_2$. Typical results regarding this issue are the well-known Schur theorem (1921) stating that every weakly convergent sequence of elements of the space ℓ_1 of all summable sequences is norm convergent, or the Grothendieck theorem (1953) asserting that the weak* convergence in the dual of the space ℓ_∞ of all bounded sequences implies the weak convergence. The latter result will be our starting point.

We say that a Banach space X is *Grothendieck* if every weak* convergent sequence $\langle x_n^* \in X\colon\ n \in \omega \rangle$ of continuous functionals on X is weakly convergent (I'll explain what it means!). Beside ℓ_∞, many Banach spaces have been recognised to be Grothendieck, e.g. the space H^∞ of bounded analytic functions on the unit disc (Bourgain 1983), or von Neumann algebras (Pfitzner 1994). Moreover, Schachermayer (1982) and Cembranos (1984) proved that an infinite-dimensional space $C(K)$ of continuous real-valued functions on a compact space K has the Grothendieck property if and only if it does not contain any complemented copies of the space c_0 of all convergent sequences.

For a long time all known examples of infinite-dimensional Grothendieck $C(K)$-spaces had density continuum $\mathfrak{c}$. Brech (2006) using the side-by-side Sacks forcing obtained the first consistent example of a Grothendieck $C(K)$-space of density ω_1 while $\omega_1 < \mathfrak{c}$ holds. She then asked whether the existence of such an example may be a consequence of a single set-theoretic assumption (such as: $\mathfrak{p} < \mathfrak{c}$). During my talk I'll show how we can extend Brech's result to a greater class of forcings (including e.g. Silver and Miller) as well as answer her question affirmatively.

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