Almost strongly zero-dimensional spaces

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A subset $A$ of a topological space $X$ is called a $C$-set if there is a sequence $(U_n)_{n \in \omega}$ of clopen sets in $X$ such that

$$A = \bigcap_{n \in \omega} U_n.$$ 

We say that $A$ is a $C_\sigma$-set in $X$ if it is a union of a sequence of $C$-sets.

A topological space $X$ is

- strongly zero-dimensional if every zero set in $X$ is a $C$-set;
- almost strongly zero-dimensional if every zero set in $X$ is a $C_\sigma$-set.

Almost strongly zero-dimensional spaces arise naturally in questions concerning the Baire classification of $F_\sigma$-measurable functions.

**Theorem 1.** Let $X, Y$ be metrizable spaces and $Y$ is separable and disconnected. The following conditions are equivalent:

1) every $F_\sigma$-measurable function $f : X \to Y$ belongs to the first Baire class;
2) $X$ is almost strongly zero-dimensional.

Clearly, each strongly zero-dimensional space is almost strongly zero-dimensional. Moreover, every absolutely analytic separable space or countably compact metric space is almost strongly zero-dimensional iff it is strongly zero-dimensional. However, the following question is open.

**Question 1.** Do there exits a completely regular almost strongly zero-dimensional space $X$ with $\dim X > 0$?