ALMOST STRONGLY ZERO-DIMENSIONAL SPACES

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A subset A of a topological space X is called *a C-set* if there is a sequence $(U_n)_{n\in\omega}$ of clopen sets in X such that

$$A = \bigcap_{n\in\omega} U_n.$$

We say that A is *a C_σ-set in X* if it is a union of a sequence of C-sets.

A topological space X is

- *strongly zero-dimensional* if every zero set in X is a C-set;
- *almost strongly zero-dimensional* if every zero set in X is a C_σ-set.

Almost strongly zero-dimensional spaces arises naturally in questions concerning the Baire classification of F_σ-measurable functions.

Theorem 1. *Let X, Y be metrizable spaces and Y is separable and disconnected. The following conditions are equivalent:*

1) every F_σ-measurable function $f : X \to Y$ belongs to the first Baire class;

2) X is almost strongly zero-dimensional.

Clearly, each strongly zero-dimensional space is almost strongly zero-dimensional. Moreover, every absolutely analytic separable space or countably compact metric space is almost strongly zero-dimensional iff it is strongly zero-dimensional. However, the following question is open.

Question 1. *Do there exits a completely regular almost strongly zero-dimensional space X with* $\dim X > 0$*?*