Reflection cardinals of coloring of graphs

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Abstract. We consider reflection cardinals:

\[ \mathcal{R}efl_{>\delta}^{\text{col}} = \min \{ \kappa : \text{for any graph } G, \text{ if } \text{col}(G) > \delta \text{ then there is } G' \in S_{<\kappa}(G) \text{ with } \text{col}(G') > \delta \} \]

\[ \mathcal{R}efl_{>\delta}^{\text{chr}} = \min \{ \kappa : \text{for any graph } G, \text{ if } \text{chr}(G) > \delta \text{ then there is } G' \in S_{<\kappa}(G) \text{ with } \text{chr}(G') > \delta \} \]

We show that the inequality \( \mathcal{R}efl_{>\delta}^{\text{col}} \leq \mathcal{R}efl_{>\delta}^{\text{chr}} \) holds for regular cardinals \( \delta \) such that \( \delta^\delta = \delta \). Our proof is rather indirect one but for that it connects many other natural reflection cardinals with these cardinals. As an application we mention an alternative proof of some results in SH1006.