Szymon Głąb

Dense free subgroups of automorphism groups of homogeneous partially ordered sets

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ultrahomogneous structures

ultrahomogeneous structure

We say that a countable structure A is *ultrahomogeneous*, if each isomorphism between finitely generated substructures of A can be extended to an automorphism of A.

A is ultrahomogeneous iff A is a Fraïssé limit.

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Let $\mathcal{K}$ be a class of finitely generated $\mathcal{L}$-structures. $\mathcal{K}$ is called age if it has

- Hereditary property (HP): if $A \in \mathcal{K}$ and B is finitely generated substructure of A, then $B \in \mathcal{K}$.
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Age of ultrahomogeneous structure

Let $\mathcal{K}$ be a family of all finitely generated substructures of ultrahomogeneous structure A. Then $\mathcal{K}$ is an age (of A).

Fraïssé theorem

Let $\mathcal{L}$ be a countable language and $\mathcal{K}$ be a countable age of $\mathcal{L}$-structures. Then there is $\mathcal{L}$-structure A, unique up to isomorphism, such that

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Some examples of Fraïssé limits

finite linear orders

If $\mathcal{K} = \{\text{finite linear orders}\}$, then $(\mathbb{Q}, \leq)$ is a Fraïssé limit of $\mathcal{K}$.

finite graphs

Random graph $\mathbb{G}$ is a Fraïssé limit of $\mathcal{K} = \{\text{finite graphs}\}$.

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Hall's universal locally finite group is a Fraïssé limit of {finite groups}.

finite groups where every element has order 2

$\bigoplus_{i \in \mathbb{N}} \mathbb{Z}_2$ is a Fraïssé limit of {finite groups where every element has order 2}.

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Schmerl's characterization of countable homogeneous partial orders

Let $1 \leq n \leq \omega$. Let $A_n := \{0, 1, \ldots, n-1\}$. Define $<$ on A_n so that for no $x, y \in A_n$ is $x < y$. Let $B_n = A_n \times \mathbb{Q}$. Define $<$ on B_n so that $(k, p) < (m, q)$ iff $k = m$ and $p < q$. Let $C_n = B_n$ and define $<$ on C_n so that $(k, p) < (m, q)$ iff $p < q$. Finally, let $(D, <)$ be the universal countable homogeneous partially ordered set, that is a Fraïssé limit of all finite partial orders.

Schmerl, 1979

Let $(H, <)$ be a countable partially ordered set. Then $(H, <)$ is ultrahomogeneous iff it is isomorphic to one of the following:

(a) $(A_n, <)$ for $1 \leq n \leq \omega$;

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Moreover, no two of the partially ordered sets listed above are isomorphic.

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Infinite countable homogeneous partial orders are freely topologically 2-generated

A topological group G is freely topologically 2-generated if there are two elements $f, g \in G$ such that $\langle f, g \rangle$ is a dense free subgroup of G.
Basic open sets – $\{f \in \mathrm{Aut}(X) : h \subset f\}$ where h is a partial isomorphism of X.

Theorem

Let $n \leq \omega$. The following groups $\mathrm{Aut}(A_\omega) = S_\infty$, $\mathrm{Aut}(B_n)$, $\mathrm{Aut}(C_n)$ and $\mathrm{Aut}(D)$ are freely topologically 2-generated.

The case of S_∞ and $\mathrm{Aut}(\mathbb{Q}) = \mathrm{Aut}(B_1)$ was deeply investigated by Darji and Mitchell.

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Enumerate all words by w_k, all partial isomorphisms by h_k, and all elements from B_n (or C_n or D) by q_k. We will define the sequence of partial isomorphisms f_i, g_i such that

(a) $f_{i-1} \subset f_i$ and $g_{i-1} \subset g_i$;

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Infinite countable homogeneous partial orders are freely topologically 2-generated

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C_n, $n \in \mathbb{N}$ or $n = \omega$

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