ON THE SEMITOPOLOGICAL LOCALLY COMPACT $\alpha$-BICYCLIC MONOID

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For each ordinal $\alpha$ by the $\alpha$-bicyclic monoid $B_\alpha$ we mean the set $\omega^\alpha \times \omega^\alpha$ endowed with the following binary operation:

$$(a, b) \cdot (c, d) = \begin{cases} 
(a + (c - b), d), & \text{if } b \leq c; \\
(a, d + (b - c)), & \text{if } b > c.
\end{cases}$$

We prove that $\alpha$-bicyclic monoid $B_\alpha$ is algebraically isomorphic to a semigroup of all order isomorphisms between the principal upper sets of the ordinal $\omega^\alpha$ and prove that $B_{\alpha+1}$ is isomorphic to the Brook extension of the semigroup $B_\alpha$. We prove that for every ordinal $\alpha$ for every $(a, b) \in B_\alpha$ if either $a$ or $b$ is a non-limit ordinal then $(a, b)$ is an isolated point in the semitopological $B_\alpha$. We show that for every ordinal $\alpha < \omega + 1$ every locally compact semigroup topology on $B_\alpha$ is discrete. However, we construct an example of a non-discrete locally compact topology $\tau_{lc}$ on $B_{\omega+1}$ such that $(B_{\omega+1}, \tau_{lc})$ is a topological inverse semigroup. Also, for every positive integer $n$ we describe all locally compact topologies on the semitopological $B_n$. In particular we show that there exist exactly $n$ distinct locally compact topologies on the semitopological $n$-bicyclic monoid $B_n$. 

1