

Answer to a question of Rosłanowski and Shelah

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In [2] Rosłanowski and Shelah proved that both the Cantor group (2^ω) and \mathbb{R} admit a null but non-meager subgroup. Conversely, they showed that it is independent of *ZFC* whether there exists a meager, but not-null subgroup. They asked whether these hold for arbitrary (non-discrete) locally compact groups. The goal of this talk is to present the solution.

First, we construct a null, but non-meager subgroups in the case of second countable Lie groups, and the inverse limits of countable sequences of finite groups. Moreover using the solution for Hilbert's fifth problem, we reduce the general case to the aforementioned two cases.

It is known that an old theorem of Friedman implies that in the Cohen model every meager subgroup of 2^ω is null [1]. It turns out, using this and techniques similar to those needed for the opposite direction, that in the Cohen model a meager subgroup of an arbitrary non-discrete locally compact group is always null.

References

- [1] Burke, M. R.: *A theorem of Friedman on rectangle inclusion and its consequences*. Note of March 7, 1991.
- [2] Rosłanowski, A., Shelah, S.: *Small-large subgroups of the reals*. arXiv:1605.02261, 2016.
- [3] Tao, Terence: *Hilbert's Fifth Problem, and Related topics*. Graduate Studies in Mathematics, Vol. 153, American Mathematical Society, 2014.