

Luzin π -bases and the foliage hybrid operation

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The aim of this talk is to present the foliage hybrid operation. This operation modifies a foliage tree by using a family of special foliage trees. The notions of a *foliage tree* and the *foliage hybrid operation* were recently introduced in [1], where they were used as tool to build a Luzin π -base for a subspace of a space that already has a Luzin π -base.

The notion of a Luzin π -base was introduced in [2] and a brief introduction to spaces that have a Luzin π -base can be found in [1]. Let us denote by LPB the class of spaces with a Luzin π -base, here are some examples of spaces in this class:

- The Baire space ${}^\omega\omega$, the Sorgenfrey line \mathcal{S} , the irrational Sorgenfrey line \mathcal{I} ;
- if $X_\alpha \in \{\omega\omega, \mathcal{S}, \mathcal{I}\}$ and $0 < |A| \leq \aleph_0$, then $\prod_{\alpha \in A} X_\alpha \in \text{LPB}$;
- if $X \in \text{LPB}$, then $X \times {}^\omega\omega \in \text{LPB}$;
- if $X_\alpha \in \text{LPB}$ and $0 < |A| \leq \aleph_0$, then $\bigoplus_{\alpha \in A} X_\alpha \in \text{LPB}$;
- if $X \in \text{LPB}$ and $F \subseteq X$ is a σ -compact, then $X \setminus F \in \text{LPB}$.

Every space with a Luzin π -base shares many good properties of the Baire space: each space in LPB has a weak topology homeomorphic to ${}^\omega\omega$, and every space in LPB can be mapped onto an arbitral Polish (that is, separable complete metrizable) space by a continuous open map.

References

- [1] M. Patrakeev, The complement of a σ -compact subset of a space with a Luzin π -base also has a Luzin π -base, preprint, <http://arxiv.org/abs/1512.02458>
- [2] M. Patrakeev, Metrizable images of the Sorgenfrey line, *Topology Proceedings* vol.45 (2015) 253-269.

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