Radek Honzik: The tree property at the double successor of a strong limit singular cardinal with a larger gap

Abstract, Hejnice, February 5, 2016.

Suppose κ is a regular cardinal. We write $\mathrm{TP}(\kappa)$ to indicate that there is no Aronszajn tree on κ, i.e. every κ-tree has a branch of size κ (κ has the *tree property*).

We will present the context, motivation and very few details regarding the proof of the following theorem:

Theorem (Friedman, Honzik, Stejskalová, 2015)**.** *Suppose κ is a supercompact cardinal, and $\lambda > \kappa$ is a weakly compact cardinal. Then there is a forcing notion $\mathbb{R}$ such that in $V^{\mathbb{R}}$:*

- *κ is a singular strong limit cardinal with cofinality ω, $\kappa^{++} = \lambda$,*
- *$2^\kappa = \kappa^{+++}$,*
- *$\mathrm{TP}(\kappa^{++})$.*

Note that κ^{+++} is used for simplicity, any reasonable $\mu \geq \kappa^{++}$ is possible.

This theorem extends a result by Foreman from 1996 who proved the above with $2^\kappa = \kappa^{++}$.

More details are in a paper available on logika.ff.cuni.cz/radek.