Some new coanalytic complete collections of continua in cubes

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C property

Definitions (B.E. Wilder, 1968)

- A space X is said to have **property C** if X has at least three points and for any distinct $a, b, c \in X$ there exists a continuum $K \subseteq X$ containing a and exactly one of the points b, c.
- If every non-degenerate subcontinuum of X has property C, then X is said to have property **C hereditarily.**
- A **C-continuum** is a continuum that has property C. A CH-continuum is a continuum that has property C hereditarily.

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- Topological sine curve $S = \overline{\{(x, \sin(\frac{1}{x})) : x \in (0, 1]\}}$ is not a C-continuum.
- However, $S \times [0, 1]$ is a C-continuum.

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- A continuum X is called **unicoherent** if for any subcontinua $A, B \subseteq X$, $A \cup B = X \Rightarrow A \cap B$ is connected.
- X is called **hereditarily unicoherent** if every subcontinuum of X is unicoherent.
- A continuum X is **irreducible** if it is irreducible between some two of its points, i.e. for some points $x, y \in X$, X does not contain a proper subcontinuum containing x, y.

Question: is there a hereditarily unicoherent C-continuum that is not arcwise connected?

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Characterisations

We have the following characterisations using the property C.

Theorem

1. *An irreducible C-continuum is an arc. [B.E.Wilder, 1968]*
2. *A homogeneous C-continuum is a simple closed curve. [B.E.Wilder, 1992]*

Now from these we get the following fact:

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Dendroids

Definition

A **dendroid** is an arcwise connected and hereditarily unicoherent continuum.

From the previous fact we may notice the following characterisation of dendroids:

Fact

A space X is a dendroid $\iff$ X is a hereditarily unicoherent C-continuum.

Notation:
If X is a space then $2^X, C(X)$ denote the hyperspaces of all compact subsets of X and all subcontinua of X respectively. Let $I, \mathfrak{C}$ denote the unit interval $[0, 1]$ and the Cantor set respectively.

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Coanalytic completeness

Definition

Let X be a space.

- A set $A \subseteq X$ is **coanalytic hard** if for any space Y and any set $B \in \Pi^1_1(Y)$ there exists a function $f : Y \longrightarrow X$ such that $f^{-1}[A] = B$.
- A coanalytic set $A \subseteq X$ that is coanalytic hard is called **coanalytic complete**.

The Hurewicz set $\mathcal{H}$ may be defined as follows:

$$\mathcal{H} = \{A \in 2^{\mathfrak{C}} : (\forall x \in A)\text{for almost all } n \in \mathbb{N}, x(n) = 0\}$$

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Coanalytic complete classes of continua

Examples

It was proved that the following classes of continua are coanalytic complete:

- hereditarily decomposable continua in $I^n, n \in \{2, 3, \ldots, \infty\}$ [U.B. Darji, 2000];
- strongly countable dimensional continua in I^∞;
- continua in I^2 which do not contain an arc [these 2 are due to P. Krupski, 2003];
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Main result

Theorem

The set of all C-subcontinua of a cube I^n, $n \in \{2, 3, \ldots, \infty\}$ is coanalytic complete in $C(I^n)$.

Proof.

We can see that the set of all C-subcontinua in I^n is coanalytic when we write the formula defining it:

$$\mathcal{C} = \{K \in I^n : (\forall x, y, z)(x \neq y, y \neq z, z \neq x \Rightarrow$$
$$\Rightarrow (\exists L \in C(K)) x \in L \wedge ((y \notin L, z \in L) \vee (y \in L, z \notin L)))\}$$

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Coanalytic hardness

Proof.

To show that $\mathcal{C}$ is coanalytic hard, we use the fact that $\mathcal{H}$ is coanalytic complete. Therefore it is enough to construct a continuous function $f : 2^{\mathfrak{C}} \longrightarrow C(I^n)$ such that $f^{-1}[\mathcal{C}] = \mathcal{H}$. First we will construct a continuous function $f' : \mathfrak{C} \longrightarrow C(I^n)$. The aim for f' is to satisfy:

1. $(\forall x, y \in \mathfrak{C}) x \neq y \Rightarrow f'(x) \cap f'(y) = \{0\} \times I$;
2. $f'(x)$is arcwise connected $\iff x(n) = 0$ for almost all n;
3. $f'(x)$is (almost) a topological sine curve $\iff x(n) = 1$ for infinitely many n.

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Idea of construction:
For a sequence $x \in \mathfrak{C}$, let

$$f'(x) = \bigcap_{k \in \mathbb{N}} \{\text{'Strips' } S_{x \restriction k} \text{ in the square}\} \cup (\{0\} \times I).$$

Strip $S_{x \restriction k}$ is defined so that it makes as many 'turns' as there are $i \leq k$ so that $x(i) = 1$. That way, if there are infinitely many ones in x, then $f'(x)$ makes infinitely many 'turns', so it is (almost) a topological sine curve.
However, if almost all terms of x are 0, then at some point N strips $S_{x \restriction N}$ 'stop turning' and only become thinner, therefore in this case $f'(x)$ is arcwise connected.

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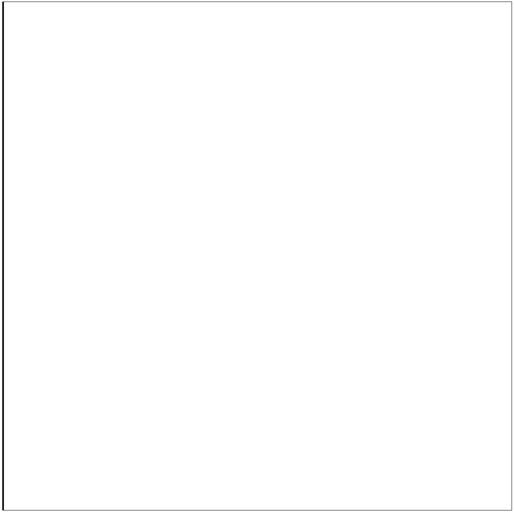

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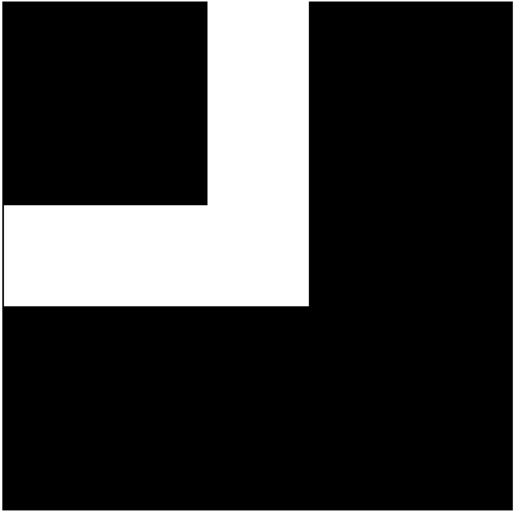

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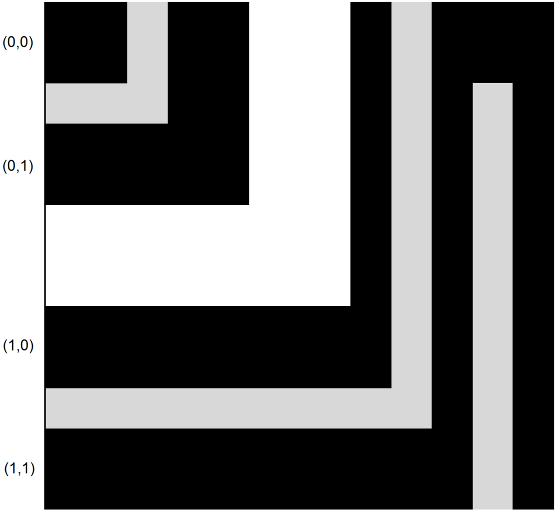

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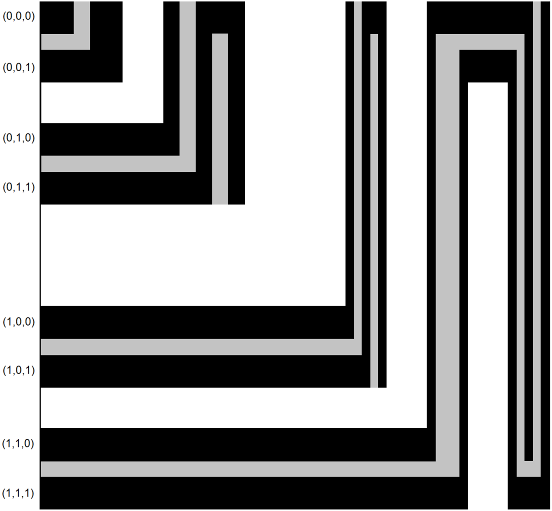

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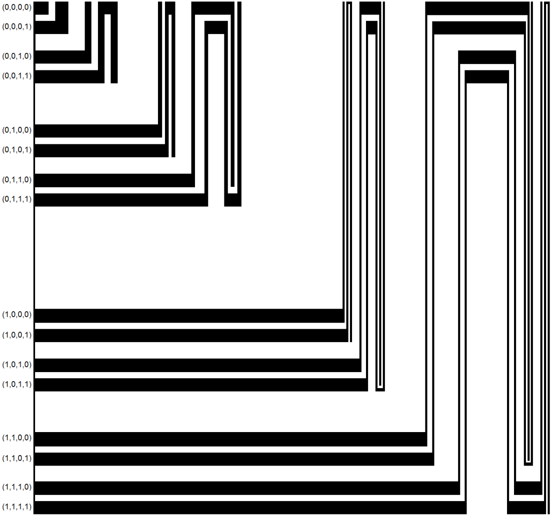

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Having defined f', let $f : 2^{\mathfrak{c}} \longrightarrow C(I^n)$ be defined as:

$$f(A) = \bigcup f'[A] = \bigcap_{k \in \mathbb{N}} \bigcup_{x \in A} S_{x \restriction k} \cup (\{0\} \times I)$$

Now we may verify that:

- for A compact $f(A)$ is a continuum;
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Corollaries

This proof also yields that:

Theorem

1. *the set of CH-continua in I^n, $n \in \{2, 3, \ldots, \infty\}$ is coanalytic hard;*
2. *the set of dendroids in I^n is coanalytic hard.*

Combining this with the fact that dendroids are hereditarily unicoherent C-continua, we get the following:

Corollary

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Corollaries

This proof also yields that:

Theorem

1. *the set of CH-continua in* $I^n, n \in \{2, 3, \ldots, \infty\}$ *is coanalytic hard;*
2. *the set of dendroids in* I^n *is coanalytic hard.*

Combining this with the fact that dendroids are hereditarily unicoherent C-continua, we get the following:

Corollary

Set of all dendroids in $I^n, n \in \{2, 3, \ldots, \infty\}$ *is coanalytic complete.*

Thank you for your attention

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