Abstract Well and Better Quasi-Orders

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Better quasi-order theory uses Ramsey theory to show that particular orders have no infinite antichains or descending sequences, when it otherwise may be difficult to do so. The Ramsey methods come from the underlying use of the Ramsey space of infinite sequences $\mathbb{N}^{[\infty]}$, which is implicitly used in the definitions of WQO and BQO.

What happens to the notions of WQO and BQO when we instead use a different Ramsey space? Investigating this question has lead to a new way to look at the notions of well and better quasi-order and a new way to view cirtain Ramsey spaces.

We define the notions $(\mathcal{R})$-WQO and $(\mathcal{R})$-BQO for a Ramsey space $\mathcal{R}$, then look at the quasi-order of non-persistent trees of cardinality $\aleph_1$ as studied by Todorčević and Väänänen in [1]. We show that this quasi-order is $(\mathcal{R})$-BQO for a particular Ramsey space $\mathcal{R}$, even though it has been shown to have antichains of size $2^{\aleph_1}$.

References

[1] S. Todorčević and J. Väänänen, *Trees and Ehrenfeucht-Fraïssé games.* Ann. Pure Appl. Logic 100(1-3) (1999), 69-97.