UPPER NAMIOKA PROPERTY OF MULTI-VALUED MAPPINGS

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For topological spaces X and Y let $LU(X,Y)$ stands for the collection of all multi-valued mappings $F : X \times Y \to [0,1]$ which are lower semi-continuous with respect to the first variable and upper semi-continuous with respect to the second one.

Theorem. [G. Debs, [1]] *Let X be a Baire space, Y be a second countable space and $F \in LU(X,Y)$ be a compact-valued mapping. Then there exists a dense in X G_δ-set $A \subseteq X$ such that F is jointly upper semi-continuous at each point of the set $A \times Y$.*

This result is a generalization of the theorem on Namioka property of separately continuous function defined on a product of a Baire space and a metrizable compact space. Therefore, it is actual to generalize the results on Namioka and co-Namioka spaces (see [3]) on the case of compact-valued mappings.

A compact-valued mapping $F \in LU(X,Y)$ has *the upper Namioka property* if there exists a dense in X G_δ-set $A \subseteq X$ such that F is jointly upper semi-continuous at every point of $A \times Y$.

A topological (Baire) space X is called *upper Namioka*, if for every compact space Y every compact-valued mapping $F \in LU(X,Y)$ has the upper Namioka property.

A compact space Y is called *upper co-Namioka*, if for every Baire space X every compact-valued mapping $F \in LU(X,Y)$ has the upper Namioka property.

The following questions naturally arise.

Question 1. *Which of Namioka spaces are upper Namioka?*

Question 2. *Which of co-Namioka spaces are upper co-Namioka?*

Theorem 1. *Let X be a T_1-space. Then the following conditions are equivalent:*

(i) *X is upper Namioka space;*

(ii) *the set A of all isolated points of X is dense in X.*

Theorem 2.

1. Every subset of upper co-Namioka space is separable.

2. Every well-ordered upper co-Namioka compact space is metrizable.

3. There exists a family $(Y_s : s \in S)$ of upper co-Namioka spaces Y_s such that the product $Y = \prod\limits_{s\in S} Y_s$ is not upper co-Namioka.

4. Every upper co-Namioka Valdivia compact space is metrizable.

5. Let Y be a linearly ordered compact space such that Y^2 is upper co-Namioka. Then Y is metrizable.

6. The double arrow space is not upper co-Namioka space.

Theorem 3. *There exist a Namioka space X, a co-Namioka space Y and a compact-valued mapping $F \in LU(X,Y)$ such that F has not the upper Namioka property.*

Theorem 4. it Let X be a metrizable Baire space and Y be a separable linearly ordered compact space. Then every compact-valued mapping $F \in LU(X,Y)$ has the upper Namioka property.

Question 3. *Does there exist a non-metrizable (linearly ordered) upper co-Namioka space?*

Question 4. *Is it true that the product of finite (countable) family of upper co-Namioka spaces is upper co-Namioka?*

References

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