Cardinal characteristic of ideals and three sequence selection principles

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Blass A.: *Combinatorial cardinal characteristics of the Continuum* in: Foreman M., Kanamori A. (eds) Handbook of Set Theory. Springer, Dordrecht (2010), 24–27.

Slalom

A slalom is a function s assigning to each $n \in \omega$ a set $s(n) \subseteq \omega$ of cardinality n.

A real goes through slalom

We say that a real $x \in {}^{\omega}\omega$ goes through slalom s if $x(n) \in s(n)$ for all but finitely many $n \in \omega$.

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$$\mathrm{add}(\mathcal{N}) = \min\{\kappa;\ (\exists \mathcal{T} \subseteq {}^{\omega}\omega, |\mathcal{T}| = \kappa)(\forall \text{ slalom } s)(\exists \varphi \in \mathcal{T})\ \neg(\varphi \text{ goes through } s)\}$$

- if $|\mathcal{T}| < \mathrm{add}(\mathcal{N})$ then there is a slalom s such that every $\varphi \in \mathcal{T}$ goes through s

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A family $\mathcal{K} \subseteq \mathcal{P}(\omega)$ is called an ideal if

a) $B \in \mathcal{K}$ for any $B \subseteq A \in \mathcal{K}$,

b) $A \cup B \in \mathcal{K}$ for any $A, B \in \mathcal{K}$,

c) $\mathrm{Fin} = [\omega]^{<\omega} \subseteq \mathcal{K}$,

d) $\omega \notin \mathcal{K}$.

$\mathcal{I}$, $\mathcal{J}$, $\mathcal{K}$ are ideals in the following.

$\mathcal{K} \subseteq \mathcal{P}(\omega)$ $\qquad$ $\mathcal{K}^+ = \mathcal{P}(\omega) \setminus \mathcal{K}$

$\mathcal{A} \subseteq \mathcal{P}(\omega)$ $\qquad$ $\mathcal{A}^d = \{A \subseteq \omega;\ \omega \setminus A \in \mathcal{A}\}$

$\mathcal{F} \subseteq \mathcal{P}(\omega)$ is a filter if $\mathcal{F}^d$ is an ideal

A maximal filter $\mathcal{U} \subseteq \mathcal{P}(\omega)$ is called an ultrafilter.

$\mathcal{A}$-slalom, $\mathcal{A} \subseteq \mathcal{P}(\omega)$

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A real $\mathcal{J}$-goes through $\mathcal{A}$-slalom

A function $\varphi \in {}^{\omega}\omega$ $\mathcal{J}$-goes through $\mathcal{A}$-slalom s if $\{n : \varphi(n) \in s(n)\} \in \mathcal{J}^d$.

$\{n : \varphi(n) \in s(n)\} \in \mathcal{J}^d$ if and only if $\{n : \varphi(n) \in \omega \setminus s(n)\} \in \mathcal{J}$

φ goes through $\mathcal{A}$-slalom s = φ Fin-goes through $\mathcal{A}$-slalom s

Šupina J.: *Ideal* QN-*spaces*, J. Math. Anal. Appl. 434 (2016) 477-491.

$$\lambda(\mathcal{I},\mathcal{J}) = \min\{\kappa;\ (\exists \mathcal{R}, |\mathcal{R}| = \kappa)(\forall \varphi \in {}^{\omega}\omega)(\exists\, \mathcal{I}^d\text{-slalom } s \in \mathcal{R})\ \neg(\varphi\ \mathcal{J}\text{-goes through } s)\}$$

- if $|\mathcal{R}| < \lambda(\mathcal{I},\mathcal{J})$ then there is a function $\varphi \in {}^{\omega}\omega$ which $\mathcal{J}$-goes through all $\mathcal{I}^d$-slaloms in $\mathcal{R}$

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- if $\mathcal{I}_1\le_K\mathcal{I}_2$ and $\mathcal{J}_1\le_{KB}\mathcal{J}_2$ then $\lambda(\mathcal{I}_2,\mathcal{J}_1)\le\lambda(\mathcal{I}_1,\mathcal{J}_2)$
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Theorem (V. Šottová–J.Š.)

Let $\mathcal{I}$ have no $\mathcal{J}$-pseudounion. Then

$$\min\{\mathtt{cov}^*(\mathcal{I}), \mathfrak{b}\} = \lambda(\mathcal{I}, \mathrm{Fin}) \le \lambda(\mathcal{I},\mathcal{J}) \le \min\{\mathtt{cov}^*(\mathcal{I},\mathcal{J}), \mathfrak{b}_{\mathcal{J}}\}.$$

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& \mathfrak{p} \arrow[u] &
\end{tikzcd}$$

$\lambda(\mathcal{I},\mathcal{J}) = \min\{\kappa;\ (\exists\mathcal{R}, |\mathcal{R}|=\kappa)(\forall\varphi\in{}^\omega\omega)(\exists\,\mathcal{I}^d$-slalom $s\in\mathcal{R})\,\neg(\varphi\,\mathcal{J}$-goes through $s)\}$

Let $\mathcal{I}$ have no $\mathcal{J}$-pseudounion. Then

$$\min\{\mathtt{cov}^*(\mathcal{I}),\mathfrak{b}\} = \lambda(\mathcal{I},\mathrm{Fin}) \le \lambda(\mathcal{I},\mathcal{J}) \le \min\{\mathtt{cov}^*(\mathcal{I},\mathcal{J}),\mathfrak{b}_{\mathcal{J}}\}.$$

- A set B is said to be a $\mathcal{J}$**-pseudounion** of the family $\mathcal{A}$ if $\omega\setminus B\in\mathcal{J}^+$ and if $A\subseteq^{\mathcal{J}} B$ for any $A\in\mathcal{A}$.

- If $\mathcal{I}$ has no $\mathcal{J}$-pseudounion then

$$\begin{aligned}\mathtt{cov}^*(\mathcal{I},\mathcal{J}) &= \min\{|\mathcal{A}|,\ \mathcal{A}\subseteq\mathcal{I}\wedge(\forall S\in\mathcal{J}^+)(\exists A\in\mathcal{A})\ S\cap A\in\mathcal{J}^+\}\\ &= \min\{|\mathcal{A}|,\ \mathcal{A}\subseteq\mathcal{I}\wedge\mathcal{A}\text{ does not have a }\mathcal{J}\text{-pseudounion}\}\\ &= \min\{|\mathcal{A}|,\ \mathcal{A}\subseteq\mathcal{I}^d\wedge\mathcal{A}\text{ does not have a }\mathcal{J}\text{-pseudointersection}\}.\end{aligned}$$

- If $\mathcal{I}_1\subseteq\mathcal{I}_2$ and $\mathcal{J}_1\subseteq\mathcal{J}_2$ then $\mathtt{cov}^*(\mathcal{I}_2,\mathcal{J}_1)\le\mathtt{cov}^*(\mathcal{I}_1,\mathcal{J}_2)$.
- $\mathtt{cov}^*(\mathcal{I}) = \mathtt{cov}^*(\mathcal{I},\mathrm{Fin})$

$\lambda(\mathcal{I},\mathcal{J}) = \min\{\kappa;\ (\exists\mathcal{R}, |\mathcal{R}|=\kappa)(\forall\varphi\in{}^{\omega}\omega)(\exists\,\mathcal{I}^d$-slalom $s\in\mathcal{R})\ \neg(\varphi\ \mathcal{J}$-goes through $s)\}$

Let $\mathcal{I}$ have no $\mathcal{J}$-pseudounion. Then

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- If $\mathtt{cov}^*(\mathcal{I}) \ge \mathfrak{b}$ then $\mathfrak{b} \le \lambda(\mathcal{I},\mathcal{J}) \le \mathfrak{b}_{\mathcal{J}}$.

Farkas B. and Soukup L., *More on cardinal invariants of analytic P-ideals*, Comment. Math. Univ. Carolin. **50** (2009), 281–295.

- If $\mathrm{cov}^*(\mathcal{I}) \ge \mathfrak{b}$ and $\mathcal{J}$ is meager then $\lambda(\mathcal{I},\mathcal{J}) = \mathfrak{b}$.

Hrušák M., *Combinatorics of filters and ideals,* Contemp. Math. **533** (2011), 29–69.

- If $\mathcal{I}\in\{\mathrm{Fin},\mathrm{Fin}\times\mathrm{Fin},\mathcal{R},\mathrm{conv},\mathcal{ED}\}$ and $\mathcal{J}\in\{\mathrm{Fin},\mathrm{Fin}\times\mathrm{Fin},\emptyset\times\mathrm{Fin},\mathrm{Fin}\times\emptyset,\mathcal{S},\mathcal{R},\mathrm{conv},\mathcal{ED},\mathcal{Z},\mathrm{nwd}\}$ then $\lambda(\mathcal{I},\mathcal{J})=\mathfrak{b}$.
- If $\mathrm{cov}(\mathcal{N}) \ge \mathfrak{b}$ and $\mathcal{J}$ is meager then $\lambda(\mathcal{Z},\mathcal{J}) = \mathfrak{b}$.
- If $\mathrm{cov}(\mathcal{M}) \ge \mathfrak{b}$ and $\mathcal{J}$ is meager then $\lambda(\mathrm{nwd},\mathcal{J}) = \mathfrak{b}$.
- If $\mathrm{non}(\mathcal{N}) \ge \mathfrak{b}$ and $\mathcal{J}$ is meager then $\lambda(\mathcal{S},\mathcal{J}) = \mathfrak{b}$.
- If $\min\{\mathrm{cov}(\mathcal{N}),\mathrm{cov}(\mathcal{M})\} \ge \mathfrak{b}$ then $\lambda(\mathcal{Z},\mathcal{Z}) = \lambda(\mathcal{Z},\mathrm{Fin}) = \lambda(\mathrm{nwd},\mathrm{nwd}) = \lambda(\mathrm{nwd},\mathrm{Fin}) = \lambda(\mathcal{S},\mathcal{S}) = \lambda(\mathcal{S},\mathrm{Fin}) = \mathfrak{b}$.

$\lambda(\mathcal{I},\mathcal{J}) = \min\{\kappa;\ (\exists \mathcal{R}, |\mathcal{R}| = \kappa)(\forall \varphi \in {}^\omega\omega)(\exists\, \mathcal{I}^d\text{-slalom } s \in \mathcal{R})\ \neg(\varphi\ \mathcal{J}\text{-goes through } s)\}$

Let $\mathcal{I}$ have no $\mathcal{J}$-pseudounion. Then

$$\min\{\mathtt{cov}^*(\mathcal{I}), \mathfrak{b}\} = \lambda(\mathcal{I}, \mathrm{Fin}) \leq \lambda(\mathcal{I},\mathcal{J}) \leq \min\{\mathtt{cov}^*(\mathcal{I},\mathcal{J}), \mathfrak{b}_{\mathcal{J}}\}.$$

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- If $\mathrm{non}(\mathcal{N}) \geq \mathfrak{b}$ and $\mathcal{J}$ is meager then $\lambda(\mathcal{S},\mathcal{J}) = \mathfrak{b}$.
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$\lambda(\mathcal{I},\mathcal{J})=\min\{\kappa;\ (\exists\mathcal{R},|\mathcal{R}|=\kappa)(\forall\varphi\in{}^{\omega}\omega)(\exists\,\mathcal{I}^d\text{-slalom } s\in\mathcal{R})\,\neg(\varphi\,\mathcal{J}\text{-goes through } s)\}$

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$$\min\{\mathtt{cov}^*(\mathcal{I}),\mathfrak{b}\}=\lambda(\mathcal{I},\mathrm{Fin})\le\lambda(\mathcal{I},\mathcal{J})\le\min\{\mathtt{cov}^*(\mathcal{I},\mathcal{J}),\mathfrak{b}_{\mathcal{J}}\}.$$

- If $\mathtt{cov}^*(\mathcal{I})\ge\mathfrak{b}$ then $\mathfrak{b}\le\lambda(\mathcal{I},\mathcal{J})\le\mathfrak{b}_{\mathcal{J}}$.

Farkas B. and Soukup L., *More on cardinal invariants of analytic P-ideals*, Comment. Math. Univ. Carolin. **50** (2009), 281–295.

- If $\mathtt{cov}^*(\mathcal{I})\ge\mathfrak{b}$ and $\mathcal{J}$ is meager then $\lambda(\mathcal{I},\mathcal{J})=\mathfrak{b}$.

Hrušák M., *Combinatorics of filters and ideals,* Contemp. Math. **533** (2011), 29–69.

- If $\mathcal{I}\in\{\mathrm{Fin},\mathrm{Fin}\times\mathrm{Fin},\mathcal{R},\mathrm{conv},\mathcal{ED}\}$ and $\mathcal{J}\in\{\mathrm{Fin},\mathrm{Fin}\times\mathrm{Fin},\emptyset\times\mathrm{Fin},\mathrm{Fin}\times\emptyset,\mathcal{S},\mathcal{R},\mathrm{conv},\mathcal{ED},\mathcal{Z},\mathrm{nwd}\}$ then $\lambda(\mathcal{I},\mathcal{J})=\mathfrak{b}$.
- If $\mathrm{cov}(\mathcal{N})\ge\mathfrak{b}$ and $\mathcal{J}$ is meager then $\lambda(\mathcal{Z},\mathcal{J})=\mathfrak{b}$.
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$\lambda(\mathcal{I},\mathcal{J}) = \min\{\kappa;\ (\exists \mathcal{R}, |\mathcal{R}| = \kappa)(\forall \varphi \in {}^{\omega}\omega)(\exists\, \mathcal{I}^d\text{-slalom } s \in \mathcal{R})\ \neg(\varphi\ \mathcal{J}\text{-goes through } s)\}$

Let $\mathcal{I}$ have no $\mathcal{J}$-pseudounion. Then

$$\min\{\mathtt{cov}^*(\mathcal{I}), \mathfrak{b}\} = \lambda(\mathcal{I}, \mathrm{Fin}) \leq \lambda(\mathcal{I},\mathcal{J}) \leq \min\{\mathtt{cov}^*(\mathcal{I},\mathcal{J}), \mathfrak{b}_{\mathcal{J}}\}.$$

- If $\mathtt{cov}^*(\mathcal{I}) \geq \mathfrak{b}$ then $\mathfrak{b} \leq \lambda(\mathcal{I},\mathcal{J}) \leq \mathfrak{b}_{\mathcal{J}}$.

Farkas B. and Soukup L., *More on cardinal invariants of analytic P-ideals*, Comment. Math. Univ. Carolin. **50** (2009), 281–295.

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Hrušák M., *Combinatorics of filters and ideals,* Contemp. Math. **533** (2011), 29–69.

- If $\mathcal{I} \in \{\mathrm{Fin}, \mathrm{Fin}\times\mathrm{Fin}, \mathcal{R}, \mathrm{conv}, \mathcal{ED}\}$ and $\mathcal{J} \in \{\mathrm{Fin}, \mathrm{Fin}\times\mathrm{Fin}, \emptyset\times\mathrm{Fin}, \mathrm{Fin}\times\emptyset, \mathcal{S}, \mathcal{R}, \mathrm{conv}, \mathcal{ED}, \mathcal{Z}, \mathrm{nwd}\}$ then $\lambda(\mathcal{I},\mathcal{J}) = \mathfrak{b}$.
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- If $\mathrm{non}(\mathcal{N}) \geq \mathfrak{b}$ and $\mathcal{J}$ is meager then $\lambda(\mathcal{S},\mathcal{J}) = \mathfrak{b}$.
- If $\min\{\mathrm{cov}(\mathcal{N}), \mathrm{cov}(\mathcal{M})\} \geq \mathfrak{b}$ then $\lambda(\mathcal{Z},\mathcal{Z}) = \lambda(\mathcal{Z},\mathrm{Fin}) = \lambda(\mathrm{nwd},\mathrm{nwd}) = \lambda(\mathrm{nwd},\mathrm{Fin}) = \lambda(\mathcal{S},\mathcal{S}) = \lambda(\mathcal{S},\mathrm{Fin}) = \mathfrak{b}$.

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Let $\mathcal{I}$ have no $\mathcal{J}$-pseudounion. Then

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- If $\mathtt{cov}^*(\mathcal{I})\ge\mathfrak{b}$ then $\mathfrak{b}\le\lambda(\mathcal{I},\mathcal{J})\le\mathfrak{b}_\mathcal{J}$.

Farkas B. and Soukup L., *More on cardinal invariants of analytic P-ideals*, Comment. Math. Univ. Carolin. **50** (2009), 281–295.

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$\lambda(\mathcal{I},\mathcal{J}) = \min\{\kappa;\ (\exists\mathcal{R}, |\mathcal{R}| = \kappa)(\forall\varphi \in {}^{\omega}\omega)(\exists\,\mathcal{I}^{d}$-slalom $s \in \mathcal{R})\ \neg(\varphi\ \mathcal{J}$-goes through $s)\}$

Let $\mathcal{I}$ have no $\mathcal{J}$-pseudounion. Then

$$\min\{\mathtt{cov}^*(\mathcal{I}), \mathfrak{b}\} = \lambda(\mathcal{I}, \mathrm{Fin}) \leq \lambda(\mathcal{I},\mathcal{J}) \leq \min\{\mathtt{cov}^*(\mathcal{I},\mathcal{J}), \mathfrak{b}_{\mathcal{J}}\}.$$

- If $\mathtt{cov}^*(\mathcal{I}) \leq \mathfrak{b}$ then $\lambda(\mathcal{I}, \mathrm{Fin}) = \mathtt{cov}^*(\mathcal{I})$.
- If $\mathtt{cov}^*(\mathcal{I}) \geq \mathfrak{b}$ then $\lambda(\mathcal{I}, \mathrm{Fin}) = \mathfrak{b}$.
- $\mathfrak{p} = \min\{\kappa;\ (\exists\mathcal{I}$ an ideal$)\ \lambda(\mathcal{I}, \mathrm{Fin}) = \kappa\}$.
- There is an ultrafilter $\mathcal{U}$ such that $\mathfrak{p} = \lambda(\mathcal{U}, \mathrm{Fin})$.
- If $\mathfrak{b} = \mathfrak{c}$ then $\lambda(\mathcal{I}, \mathrm{Fin}) = \mathtt{cov}^*(\mathcal{I})$.
- If $\mathfrak{b} < \mathtt{cov}^*(\mathcal{I})$ then $\lambda(\mathcal{I}, \mathrm{Fin}) < \mathtt{cov}^*(\mathcal{I})$.

$\lambda(\mathcal{I},\mathcal{J}) = \min\{\kappa;\ (\exists \mathcal{R}, |\mathcal{R}| = \kappa)(\forall \varphi \in {}^\omega\omega)(\exists\, \mathcal{I}^d$-slalom $s \in \mathcal{R})\ \neg(\varphi\ \mathcal{J}$-goes through $s)\}$

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- If $\mathcal{I}$ is not a weak P-ideal (i.e. $\mathrm{Fin}\times\mathrm{Fin} \le_K \mathcal{I}$) then $\lambda(\mathcal{I},\mathrm{Fin}) = \mathtt{cov}^*(\mathcal{I})$.

Brendle J., Shelah S., *Ultrafilters on ω – their ideals and their cardinal characteristics,* Trans. Amer. Math. Soc. **351** (1999), 2643–2674.

Proposition (R.M. Canjar–P. Nyikos–J. Ketonen)

If $\mathcal{U}$ is not a P-point then $\mathtt{cov}^*(\mathcal{U}) \le \mathfrak{b}$.

$$\mathcal{A}_1 \times \mathcal{A}_2 = \{A \subseteq M_1 \times M_2;\ \{n;\ \{m;\ (n,m) \in A\} \notin \mathcal{A}_2\} \in \mathcal{A}_1\}$$

$\lambda(\mathcal{I},\mathcal{J}) = \min\{\kappa;\ (\exists\mathcal{R}, |\mathcal{R}| = \kappa)(\forall\varphi \in {}^{\omega}\omega)(\exists\,\mathcal{I}^d$-slalom $s \in \mathcal{R})\ \neg(\varphi\,\mathcal{J}$-goes through $s)\}$

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All spaces are assumed to be Hausdorff and infinite.

Diagrams hold for perfectly normal space.

All covers are assumed to be countable.

Scheepers M., *Combinatorics of open covers I: Ramsey theory,* Topology Appl. **69** (1996), 31–62.

$\mathsf{S}_1(\Gamma, \Gamma)$-property

X possesses the property $\mathsf{S}_1(\Gamma, \Gamma)$ if for every sequence $\langle \mathcal{U} : n \in \omega \rangle$ of open γ-covers there exist sets $U_n \in \mathcal{U}_n$ such that $\langle U_n;\ n \in \omega \rangle$ is a γ-cover.

- γ-cover $\langle U_n;\ n \in \omega \rangle$ - every $x \in X$ lies in all but finitely many members of $\langle U_n;\ n \in \omega \rangle$, $\Gamma(X)$ or Γ denote the family of all countable open γ -covers

Just W., Miller A.W., Scheepers M. and Szeptycki P.J., *Combinatorics of open covers* II, Topology Appl. **73** (1996), 241–266.

Scheepers M., *A sequential property of $C_p(X)$ and a covering property of Hurewicz,* Proc. Amer. Math. Soc. **125** (1997), 2789–2795.

- Tychonoff $\mathsf{S}_1(\Gamma, \Gamma)$-space is zero-dimensional
- any $\mathsf{S}_1(\Gamma, \Gamma)$-subset of a metric separable space is perfectly meager
- perfectly normal $\mathsf{S}_1(\Gamma, \Gamma)$-space has Hurewicz property
- $\mathrm{non}(\mathsf{S}_1(\Gamma, \Gamma)) = \mathfrak{b}$
- $\mathfrak{h} \leq \mathrm{add}(\mathsf{S}_1(\Gamma, \Gamma)) \leq \mathfrak{b}$
- any γ-set is an $\mathsf{S}_1(\Gamma, \Gamma)$-space
- $\mathfrak{b}$-Sierpiński set is an $\mathsf{S}_1(\Gamma, \Gamma)$-space (exists under $\mathfrak{b} = \mathrm{cov}(\mathcal{N}) = \mathrm{cof}(\mathcal{N})$)

Recław I., *Metric spaces not distinguishing pointwise and quasinormal convergence of real functions,* Bull. Acad. Polon. Sci. **45** (1997), 287–289.

- there exists an uncountable $\mathsf{S}_1(\Gamma, \Gamma)$-space (of cardinality $\mathfrak{p}$)

Sequence Selection Principles

Архангельский А.В. (Arkhangel'skiĭ A.V.), *Спектр частот топологического пространства и классификация пространств*, ДАН СССР, **206**:2 (1972), 265–268. English translation *The frequency spectrum of a topological space and the classification of spaces,* Soviet Math. Dokl. **13** (1972), 1185–1189.

A topological space Y is (α_4)-space if for any sequence $\langle S_n : n \in \omega \rangle$ of sequences converging to a point $y \in Y$, there exists a sequence S converging to y such that $S_n \cap S \neq \emptyset$ for infinitely many $n \in \omega$.

$\boxed{\mathrm{S}_1(\Gamma_y, \Gamma_y)}$

Fremlin D.H., *Sequential convergence in $\mathrm{C}_p(X)$,* Comment. Math. Univ. Carolin. **35** (1994), 371–382.

Scheepers M., *A sequential property of $\mathrm{C}_p(X)$ and a covering property of Hurewicz,* Proc. Amer. Math. Soc. **125** (1997), 2789–2795.

A topological space Y has sequence selection property, if for any $x \in Y$ and for any sequence $\langle S_n : n \in \omega \rangle$ of sequences converging to x there is a sequence $\{x_n\}_{n=0}^{\infty}$ such that $x_n \to x$ and $x_n \in S_n$ for each $n \in \omega$.

A topological space $\mathrm{C}_p(X)$ has monotonic sequence selection property, if for any sequence $\langle S_n : n \in \omega \rangle$ of sequences in $\mathrm{C}_p(X)$ converging to zero monotonically there is a sequence $\{f_n\}_{n=0}^{\infty}$ such that $f_n \to 0$ and $f_n \in S_n$ for each $n \in \omega$.

$\boxed{\mathrm{S}_1(\Gamma_{\mathbf{0}}^m, \Gamma_{\mathbf{0}})}$

perfectly normal space X

L. Bukovský [2008]	M. Scheepers [1999]	M. Scheepers [1999]
M. Sakai [2009]	D.H. Fremlin [2003]	L. Bukovský, I. Reclaw and
	L. Bukovský and J. Haleš [2007]	M. Repický [2001]
		L. Bukovský and J. Haleš [2003]
X possesses SSP^*.	$\mathrm{C}_p(X)$ has $\mathrm{S}_1(\Gamma_{\mathbf{0}}, \Gamma_{\mathbf{0}})$.	$\mathrm{C}_p(X)$ has $\mathrm{S}_1(\Gamma_{\mathbf{0}}^m, \Gamma_{\mathbf{0}})$.
X is a wQN*-space.	$\to$ X is a wQN-space.	$\to$ X is an mQN-space.
X is an $\mathrm{S}_1(\Gamma, \Gamma)$-space.	X is an $\mathrm{S}_1(\Gamma_{\mathrm{sh}}, \Gamma)$-space.	X has $\mathcal{U}_{fin}(\mathcal{O}_\omega, \Gamma)$.
$\mathfrak{b}$-Sierpiński set, γ-set		compact set
	X is perfectly meager	X has count. Menger property
	X is zero-dimensional	

Das P., *Certain types of open covers and selection principles using ideals,* Houston J. Math. **39** (2013), 637–650.

Di Maio G., Kočinac Lj.D.R., *Statistical convergence in topology,* Topology Appl., **156** (2008), 28–45.

A sequence $\langle U_n : n \in \omega\rangle$ of subsets of a topological space X is said to be an **$\mathcal{I}$-γ-cover**, if for every n, $U_n \neq X$, and for every $x \in X$, the set $\{n \in \omega : x \notin U_n\}$ belongs to $\mathcal{I}$.

$\mathcal{I}$-Γ - the family of all open $\mathcal{I}$-γ-covers

$\boxed{\mathrm{S}_1(\Gamma, \mathcal{J}\text{-}\Gamma)\text{-space}}$ $\boxed{\mathrm{S}_1(\mathcal{I}\text{-}\Gamma, \mathcal{J}\text{-}\Gamma)\text{-space}}$

Bukovský L., Das P. and Šupina J., *Ideal quasi-normal convergence and related notions*, Colloq. Math. 146 (2017), 265-281.

Any γ-set is an $\mathrm{S}_1(\mathcal{I}\text{-}\Gamma, \mathcal{J}\text{-}\Gamma)$-space.

$\mathcal{I}$-pointwise convergence $\quad f_n \xrightarrow{\mathcal{I}} f$

$$(\forall x \in X)(\forall \varepsilon > 0)(\exists A \in \mathcal{I})(\forall n \in \omega)(n \notin A \to |f_n(x) - f(x)| < \varepsilon)$$

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The $\mathrm{C}_p(X)$ possesses $\mathrm{S}_1(\mathcal{I}\text{-}\Gamma_{\mathbf{0}}, \mathcal{J}\text{-}\Gamma_{\mathbf{0}})$ if for any sequence $\langle\langle f_{n,m} : m \in \omega\rangle : n \in \omega\rangle$ of sequences of continuous real functions such that $f_{n,m} \xrightarrow{\mathcal{I}} 0$ for each n, there exists a sequence $\langle m_n : n \in \omega\rangle$ such that $f_{n,m_n} \xrightarrow{\mathcal{J}} 0$.

A sequence $\langle f_n : n \in \omega\rangle$ is called $\mathcal{I}$-almost monotone sequence if $\{n : \; f_n \not\leq f_m\} \in \mathcal{I}$ for every $m \in \omega$.

$\mathrm{S}_1(\mathcal{I}\text{-}\Gamma_{\mathbf{0}}^m, \mathcal{J}\text{-}\Gamma_{\mathbf{0}})$

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Šupina J., *Ideal* QN*-spaces*, J. Math. Anal. Appl. 434 (2016) 477-491.

Lemma

Let X be a topological space. If $|X| < \lambda(\mathcal{I},\mathcal{J})$ then X is an $\mathrm{S}_1(\mathcal{I}\text{-}\Gamma,\mathcal{J}\text{-}\Gamma)$-space.

Theorem (V. Šottová–J.Š.)

Let D be a discrete topological space. Then the following statements are equivalent.

(a) *D is an $\mathrm{S}_1(\mathcal{I}\text{-}\Gamma,\mathcal{J}\text{-}\Gamma)$-space.*

(b) *$C_p(D)$ has the property $\mathrm{S}_1(\mathcal{I}\text{-}\Gamma_{\mathbf{0}},\mathcal{J}\text{-}\Gamma_{\mathbf{0}})$*

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- If $\mathfrak{p} < \mathfrak{b}$ there is an $\mathrm{S}_1(\Gamma, \Gamma)$-space which is not an $\mathrm{S}_1(\mathcal{U}\text{-}\Gamma, \Gamma)$-space.
- If $\mathtt{cov}^*(\mathcal{I}) < \mathfrak{b}$ there is an $\mathrm{S}_1(\Gamma, \Gamma)$-space which is not an $\mathrm{S}_1(\mathcal{I}\text{-}\Gamma, \Gamma)$-space.
- For any $\mathfrak{b}$-Sierpiński set S there is an ultrafilter $\mathcal{U}$ such that S is not an $\mathrm{S}_1(\mathcal{U}\text{-}\Gamma, \Gamma)$-space (but S is an $\mathrm{S}_1(\Gamma, \Gamma)$-space).
- If $\mathfrak{b} < \mathfrak{b}_{\mathcal{U}}$ then there is an $\mathrm{S}_1(\Gamma, \mathcal{U}\text{-}\Gamma)$-space which is not an $\mathrm{S}_1(\Gamma, \Gamma)$-space.

Das P. and Chandra D., *Spaces not distinguishing pointwise and $\mathcal{I}$-quasinormal convergence of real functions,* Comment. Math. Univ. Carolin. **54** (2013), 83–96.

$\mathcal{I}$-quasi-normal convergence $\quad f_n \xrightarrow{\mathcal{I}\mathrm{QN}} f$

there exists $\{\varepsilon_n\}_{n=0}^{\infty}$ $\mathcal{I}$-converging to 0 such that

$$(\forall x \in X)(\exists A \in \mathcal{I})(\forall n \in \omega)(n \notin A \to |f_n(x) - f(x)| < \varepsilon_n)$$

($\mathcal{I}$,$\mathcal{J}$)wQN-space

X is an ($\mathcal{I}$,$\mathcal{J}$)wQN-space, if for any sequence $\langle f_n : n \in \omega \rangle$ of continuous functions on X converging to zero there exists a sequence $\langle m_n : n \in \omega \rangle$ such that $f_{m_n} \xrightarrow{\mathcal{J}\mathrm{QN}} 0$.

- $\mathrm{non}((\mathcal{I}, \mathrm{Fin})\mathrm{wQN}) = \min\{\mathrm{cov}^*(\mathcal{I}), \mathfrak{b}\}$

Kwela A., *Ideal weak QN-space*, preprint.

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Thanks for Your attention!