

Cardinal characteristic of ideals and three sequence selection principles

Jaroslav Šupina
joint work with Viera Šottová

Institute of Mathematics
Faculty of Science
P.J. Šafárik University in Košice

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Blass A.: *Combinatorial cardinal characteristics of the Continuum* in: Foreman M., Kanamori A. (eds) *Handbook of Set Theory*. Springer, Dordrecht (2010), 24–27.

Slalom

A slalom is a function s assigning to each $n \in \omega$ a set $s(n) \subseteq \omega$ of cardinality n .

A real goes through slalom

We say that a real $x \in {}^\omega\omega$ goes through slalom s if $x(n) \in s(n)$ for all but finitely many $n \in \omega$.



Bartoszyński T.: *Additivity of measure implies additivity of category*, *Trans. Amer. Math. Soc.* **281** (1984), 209–213.

$$\text{add}(\mathcal{N}) = \min\{\kappa; (\exists \mathcal{T} \subseteq {}^\omega\omega, |\mathcal{T}| = \kappa)(\forall \text{slalom } s)(\exists \varphi \in \mathcal{T}) \neg(\varphi \text{ goes through } s)\}$$

► if $|\mathcal{T}| < \text{add}(\mathcal{N})$ then there is a slalom s such that every $\varphi \in \mathcal{T}$ goes through s



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A family $\mathcal{K} \subseteq \mathcal{P}(\omega)$ is called an ideal if

- a) $B \in \mathcal{K}$ for any $B \subseteq A \in \mathcal{K}$,
- b) $A \cup B \in \mathcal{K}$ for any $A, B \in \mathcal{K}$,
- c) $\text{Fin} = [\omega]^{<\omega} \subseteq \mathcal{K}$,
- d) $\omega \notin \mathcal{K}$.

$\mathcal{I}, \mathcal{J}, \mathcal{K}$ are ideals in the following.

$$\mathcal{K} \subseteq \mathcal{P}(\omega) \quad \mathcal{K}^+ = \mathcal{P}(\omega) \setminus \mathcal{K}$$

$$\mathcal{A} \subseteq \mathcal{P}(\omega) \quad \mathcal{A}^d = \{A \subseteq \omega; \omega \setminus A \in \mathcal{A}\}$$

$\mathcal{F} \subseteq \mathcal{P}(\omega)$ is a filter if \mathcal{F}^d is an ideal

A maximal filter $\mathcal{U} \subseteq \mathcal{P}(\omega)$ is called an ultrafilter.

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A real \mathcal{J} -goes through \mathcal{A} -slalom

A function $\varphi \in {}^\omega\omega$ \mathcal{J} -goes through \mathcal{A} -slalom s if $\{n : \varphi(n) \in s(n)\} \in \mathcal{J}^d$.

$\{n : \varphi(n) \in s(n)\} \in \mathcal{J}^d$ if and only if $\{n : \varphi(n) \in \omega \setminus s(n)\} \in \mathcal{J}$

φ goes through \mathcal{A} -slalom $s = \varphi$ Fin-goes through \mathcal{A} -slalom s



Šupina J.: Ideal QN-spaces, J. Math. Anal. Appl. 434 (2016) 477-491.

$\lambda(\mathcal{I}, \mathcal{J}) = \min\{\kappa; (\exists \mathcal{R}, |\mathcal{R}| = \kappa)(\forall \varphi \in {}^\omega\omega)(\exists \mathcal{I}^d\text{-slalom } s \in \mathcal{R}) \neg(\varphi \mathcal{J}\text{-goes through } s)\}$

- ▶ if $|\mathcal{R}| < \lambda(\mathcal{I}, \mathcal{J})$ then there is a function $\varphi \in {}^\omega\omega$ which \mathcal{J} -goes through all \mathcal{I}^d -slaloms in \mathcal{R}

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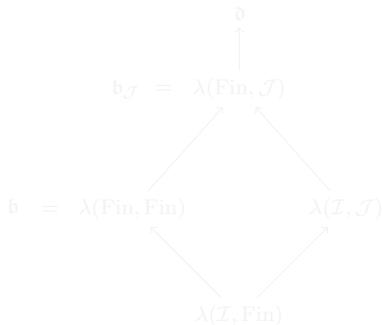
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► $\lambda(\text{Fin}, \mathcal{J}) = \mathfrak{b}_{\mathcal{J}}$



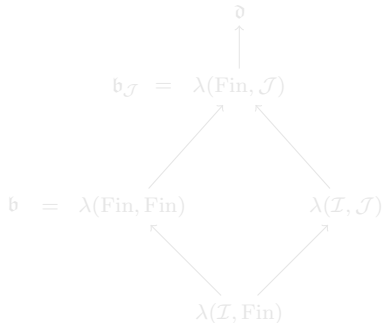
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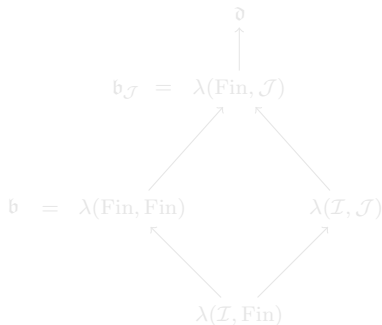


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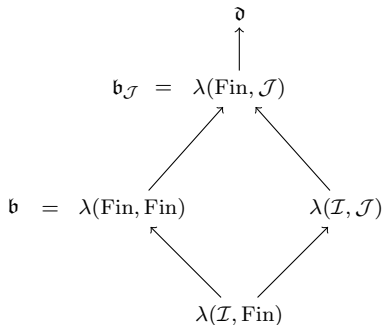


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Theorem (V. Šottová–J.Š.)

Let \mathcal{I} have no \mathcal{J} -pseudounion. Then

$$\min\{\text{cov}^*(\mathcal{I}), \mathfrak{b}\} = \lambda(\mathcal{I}, \text{Fin}) \leq \lambda(\mathcal{I}, \mathcal{J}) \leq \min\{\text{cov}^*(\mathcal{I}, \mathcal{J}), \mathfrak{b}_{\mathcal{J}}\}.$$



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\mathcal{I} is a tall ideal on ω - \mathcal{I} does not have a pseudounion

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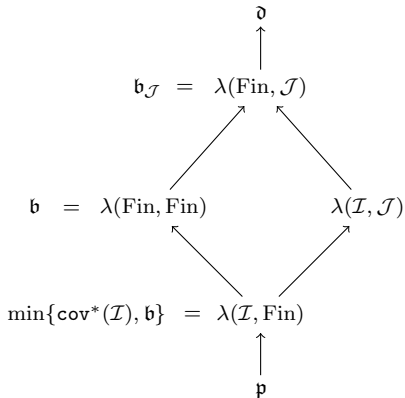
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$$\lambda(\mathcal{I}, \mathcal{J}) = \min\{\kappa; (\exists \mathcal{R}, |\mathcal{R}| = \kappa)(\forall \varphi \in {}^\omega \omega)(\exists \mathcal{I}^d\text{-slalom } s \in \mathcal{R}) \neg(\varphi \text{ } \mathcal{J}\text{-goes through } s)\}$$

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- ▶ A set B is said to be a \mathcal{J} -**pseudounion** of the family \mathcal{A} if $\omega \setminus B \in \mathcal{J}^+$ and if $A \subseteq^{\mathcal{J}} B$ for any $A \in \mathcal{A}$.
- ▶ If \mathcal{I} has no \mathcal{J} -pseudounion then

$$\begin{aligned} \text{cov}^*(\mathcal{I}, \mathcal{J}) &= \min\{|\mathcal{A}|, \mathcal{A} \subseteq \mathcal{I} \wedge (\forall S \in \mathcal{J}^+)(\exists A \in \mathcal{A}) S \cap A \in \mathcal{J}^+\} \\ &= \min\{|\mathcal{A}|, \mathcal{A} \subseteq \mathcal{I} \wedge \mathcal{A} \text{ does not have a } \mathcal{J}\text{-pseudounion}\} \\ &= \min\{|\mathcal{A}|, \mathcal{A} \subseteq \mathcal{I}^d \wedge \mathcal{A} \text{ does not have a } \mathcal{J}\text{-pseudointersection}\}. \end{aligned}$$

- ▶ If $\mathcal{I}_1 \subseteq \mathcal{I}_2$ and $\mathcal{J}_1 \subseteq \mathcal{J}_2$ then $\text{cov}^*(\mathcal{I}_2, \mathcal{J}_1) \leq \text{cov}^*(\mathcal{I}_1, \mathcal{J}_2)$.
- ▶ $\text{cov}^*(\mathcal{I}) = \text{cov}^*(\mathcal{I}, \text{Fin})$

$\lambda(\mathcal{I}, \mathcal{J}) = \min\{\kappa; (\exists \mathcal{R}, |\mathcal{R}| = \kappa)(\forall \varphi \in {}^\omega\omega)(\exists \mathcal{I}^d\text{-slalom } s \in \mathcal{R}) \neg(\varphi \text{ } \mathcal{J}\text{-goes through } s)\}$

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Farkas B. and Soukup L., *More on cardinal invariants of analytic P-ideals*, Comment. Math. Univ. Carolin. **50** (2009), 281–295.

► If $\text{cov}^*(\mathcal{I}) \geq \mathfrak{b}$ and \mathcal{J} is meager then $\lambda(\mathcal{I}, \mathcal{J}) = \mathfrak{b}$.



Hrušák M., *Combinatorics of filters and ideals*, Contemp. Math. **533** (2011), 29–69.

► If $\mathcal{I} \in \{\text{Fin}, \text{Fin} \times \text{Fin}, \mathcal{R}, \text{conv}, \mathcal{ED}\}$ and $\mathcal{J} \in \{\text{Fin}, \text{Fin} \times \text{Fin}, \emptyset \times \text{Fin}, \text{Fin} \times \emptyset, \mathcal{S}, \mathcal{R}, \text{conv}, \mathcal{ED}, \mathcal{Z}, \text{nwd}\}$ then $\lambda(\mathcal{I}, \mathcal{J}) = \mathfrak{b}$.

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- If \mathcal{I} is not a weak P-ideal (i.e. $\text{Fin} \times \text{Fin} \leq_K \mathcal{I}$) then $\lambda(\mathcal{I}, \text{Fin}) = \text{cov}^*(\mathcal{I})$.



Brendle J., Shelah S., *Ultrafilters on ω – their ideals and their cardinal characteristics*, Trans. Amer. Math. Soc. 351 (1999), 2643–2674.

Proposition (R.M. Canjar–P. Nyikos–J. Ketonen)

If \mathcal{U} is not a P-point then $\text{cov}^*(\mathcal{U}) \leq \mathfrak{b}$.

$$\mathcal{A}_1 \times \mathcal{A}_2 = \{A \subseteq M_1 \times M_2; \{n; \{m; (n, m) \in A\} \notin \mathcal{A}_2\} \in \mathcal{A}_1\}$$

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All spaces are assumed to be Hausdorff and infinite.

Diagrams hold for perfectly normal space.

All covers are assumed to be countable.



$S_1(\Gamma, \Gamma)$ -property

X possesses the property $S_1(\Gamma, \Gamma)$ if for every sequence $\langle \mathcal{U} : n \in \omega \rangle$ of open γ -covers there exist sets $U_n \in \mathcal{U}_n$ such that $\langle U_n ; n \in \omega \rangle$ is a γ -cover.

- ▶ γ -cover $\langle U_n ; n \in \omega \rangle$ - every $x \in X$ lies in all but finitely many members of $\langle U_n ; n \in \omega \rangle$, $\Gamma(X)$ or Γ denote the family of all countable open γ -covers
-



Just W., Miller A.W., Scheepers M. and Szeptycki P.J., *Combinatorics of open covers II*, Topology Appl. **73** (1996), 241–266.



Scheepers M., *A sequential property of $C_p(X)$ and a covering property of Hurewicz*, Proc. Amer. Math. Soc. **125** (1997), 2789–2795.

- ▶ Tychonoff $S_1(\Gamma, \Gamma)$ -space is zero-dimensional
 - ▶ any $S_1(\Gamma, \Gamma)$ -subset of a metric separable space is perfectly meager
 - ▶ perfectly normal $S_1(\Gamma, \Gamma)$ -space has Hurewicz property
 - ▶ $\text{non}(S_1(\Gamma, \Gamma)) = \mathfrak{b}$
 - ▶ $\mathfrak{h} \leq \text{add}(S_1(\Gamma, \Gamma)) \leq \mathfrak{b}$
 - ▶ any γ -set is an $S_1(\Gamma, \Gamma)$ -space
 - ▶ \mathfrak{b} -Sierpiński set is an $S_1(\Gamma, \Gamma)$ -space (exists under $\mathfrak{b} = \text{cov}(\mathcal{N}) = \text{cof}(\mathcal{N})$)
-



Reclaw I., *Metric spaces not distinguishing pointwise and quasnormal convergence of real functions*, Bull. Acad. Polon. Sci. **45** (1997), 287–289.

- ▶ there exists an uncountable $S_1(\Gamma, \Gamma)$ -space (of cardinality \mathfrak{p})

Sequence Selection Principles



Архангельский А.В. (Arkhangel'skii A.V.), *Спектр частот топологического пространства и классификация пространств*, ДАН СССР, **206:2** (1972), 265–268. English translation *The frequency spectrum of a topological space and the classification of spaces*, Soviet Math. Dokl. **13** (1972), 1185–1189.

A topological space Y is (α_4) -space if for any sequence $\langle S_n : n \in \omega \rangle$ of sequences converging to a point $y \in Y$, there exists a sequence S converging to y such that $S_n \cap S \neq \emptyset$ for infinitely many $n \in \omega$.

$$S_1(\Gamma_y, \Gamma_y)$$



Fremlin D.H., *Sequential convergence in $C_p(X)$* , Comment. Math. Univ. Carolin. **35** (1994), 371–382.



Scheepers M., *A sequential property of $C_p(X)$ and a covering property of Hurewicz*, Proc. Amer. Math. Soc. **125** (1997), 2789–2795.

A topological space Y has sequence selection property, if for any $x \in Y$ and for any sequence $\langle S_n : n \in \omega \rangle$ of sequences converging to x there is a sequence $\{x_n\}_{n=0}^{\infty}$ such that $x_n \rightarrow x$ and $x_n \in S_n$ for each $n \in \omega$.

A topological space $C_p(X)$ has monotonic sequence selection property, if for any sequence $\langle S_n : n \in \omega \rangle$ of sequences in $C_p(X)$ converging to zero monotonically there is a sequence $\{f_n\}_{n=0}^{\infty}$ such that $f_n \rightarrow 0$ and $f_n \in S_n$ for each $n \in \omega$.

$$S_1(\Gamma_{\mathbf{0}}^m, \Gamma_{\mathbf{0}})$$

perfectly normal space X

L. Bukovský [2008]

M. Sakai [2009]

M. Scheepers [1999]

D.H. Fremlin [2003]

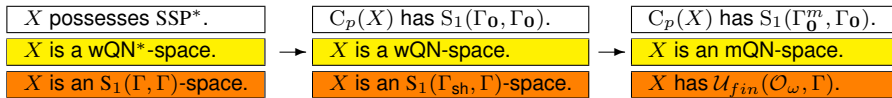
L. Bukovský and J. Haleš [2007]

M. Scheepers [1999]

L. Bukovský, I. Reclaw and

M. Repický [2001]

L. Bukovský and J. Haleš [2003]



\mathfrak{b} -Sierpiński set, γ -set

X is perfectly meager

X is zero-dimensional

compact set

X has count. Menger property



Das P., *Certain types of open covers and selection principles using ideals*, Houston J. Math. **39** (2013), 637–650.



Di Maio G., Kočinac Lj.D.R., *Statistical convergence in topology*, Topology Appl., **156** (2008), 28–45.

A sequence $\langle U_n : n \in \omega \rangle$ of subsets of a topological space X is said to be an \mathcal{I} - γ -**cover**, if for every n , $U_n \neq X$, and for every $x \in X$, the set $\{n \in \omega : x \notin U_n\}$ belongs to \mathcal{I} .

\mathcal{I} - Γ - the family of all open \mathcal{I} - γ -covers

$S_1(\Gamma, \mathcal{J}\text{-}\Gamma)$ -space

$S_1(\mathcal{I}\text{-}\Gamma, \mathcal{J}\text{-}\Gamma)$ -space



Bukovský L., Das P. and Šupina J., *Ideal quasi-normal convergence and related notions*, Colloq. Math. **146** (2017), 265-281.

Any γ -set is an $S_1(\mathcal{I}\text{-}\Gamma, \mathcal{J}\text{-}\Gamma)$ -space.

\mathcal{I} -pointwise convergence

$$f_n \xrightarrow{\mathcal{I}} f$$

$$(\forall x \in X)(\forall \varepsilon > 0)(\exists A \in \mathcal{I})(\forall n \in \omega)(n \notin A \rightarrow |f_n(x) - f(x)| < \varepsilon)$$



Bukovský L., Das P. and Šupina J., *Ideal quasi-normal convergence and related notions*, Colloq. Math. 146 (2017), 265-281.

The $C_p(X)$ possesses $S_1(\mathcal{I}\text{-}\Gamma_0, \mathcal{J}\text{-}\Gamma_0)$ if for any sequence $\langle \langle f_{n,m} : m \in \omega \rangle : n \in \omega \rangle$ of sequences of continuous real functions such that $f_{n,m} \xrightarrow{\mathcal{I}} 0$ for each n , there exists a sequence $\langle m_n : n \in \omega \rangle$ such that $f_{n,m_n} \xrightarrow{\mathcal{J}} 0$.

A sequence $\langle f_n : n \in \omega \rangle$ is called \mathcal{I} -almost monotone sequence if $\{n : f_n \not\leq f_m\} \in \mathcal{I}$ for every $m \in \omega$.

$S_1(\mathcal{I}\text{-}\Gamma_0^m, \mathcal{J}\text{-}\Gamma_0)$

The $C_p(X)$ possesses $S_1(\mathcal{I}\text{-}\Gamma_0^m, \mathcal{J}\text{-}\Gamma_0)$ if for any sequence $\langle \langle f_{n,m} : m \in \omega \rangle : n \in \omega \rangle$ of \mathcal{I} -almost monotone sequences of continuous real functions such that $f_{n,m} \xrightarrow{\mathcal{I}} 0$ for each n , there exists a sequence $\langle m_n : n \in \omega \rangle$ such that $f_{n,m_n} \xrightarrow{\mathcal{J}} 0$.



Lemma

Let X be a topological space. If $|X| < \lambda(\mathcal{I}, \mathcal{J})$ then X is an $S_1(\mathcal{I}-\Gamma, \mathcal{J}-\Gamma)$ -space.

Theorem (V. Šottová–J.Š.)

Let D be a discrete topological space. Then the following statements are equivalent.

- (a) D is an $S_1(\mathcal{I}-\Gamma, \mathcal{J}-\Gamma)$ -space.
- (b) $C_p(D)$ has the property $S_1(\mathcal{I}-\Gamma_0, \mathcal{J}-\Gamma_0)$
- (c) $C_p(D)$ has the property $S_1(\mathcal{I}-\Gamma_0^m, \mathcal{J}-\Gamma_0)$.
- (d) $|D| < \lambda(\mathcal{I}, \mathcal{J})$.

$$\blacktriangleright \text{non}(S_1(\mathcal{I}-\Gamma, \mathcal{J}-\Gamma)) = \text{non}(S_1(\mathcal{I}-\Gamma_0, \mathcal{J}-\Gamma_0)) = \text{non}(S_1(\mathcal{I}-\Gamma_0^m, \mathcal{J}-\Gamma_0)) = \lambda(\mathcal{I}, \mathcal{J})$$

$$\blacktriangleright \text{non}(S_1(\Gamma, \mathcal{J}-\Gamma)) = \text{non}(S_1(\Gamma_0, \mathcal{J}-\Gamma_0)) = \text{non}(S_1(\Gamma_0^m, \mathcal{J}-\Gamma_0)) = \mathfrak{b}_{\mathcal{J}}$$

$$\blacktriangleright \text{non}(S_1(\mathcal{I}-\Gamma, \Gamma)) = \text{non}(S_1(\mathcal{I}-\Gamma_0, \Gamma_0)) = \text{non}(S_1(\mathcal{I}-\Gamma_0^m, \Gamma_0)) = \min\{\text{cov}^*(\mathcal{I}), \mathfrak{b}\}$$



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- ▶ $\text{non}(S_1(\mathcal{I}\text{-}\Gamma, \mathcal{J}\text{-}\Gamma)) = \text{non}(S_1(\mathcal{I}\text{-}\Gamma_{\mathbf{0}}, \mathcal{J}\text{-}\Gamma_{\mathbf{0}})) = \text{non}(S_1(\mathcal{I}\text{-}\Gamma_{\mathbf{0}}^m, \mathcal{J}\text{-}\Gamma_{\mathbf{0}})) = \lambda(\mathcal{I}, \mathcal{J})$
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- ▶ If $\mathfrak{p} < \mathfrak{b}$ there is an $S_1(\Gamma, \Gamma)$ -space which is not an $S_1(\mathcal{U}\text{-}\Gamma, \Gamma)$ -space.
- ▶ If $\text{cov}^*(\mathcal{I}) < \mathfrak{b}$ there is an $S_1(\Gamma, \Gamma)$ -space which is not an $S_1(\mathcal{I}\text{-}\Gamma, \Gamma)$ -space.
- ▶ For any \mathfrak{b} -Sierpiński set S there is an ultrafilter \mathcal{U} such that S is not an $S_1(\mathcal{U}\text{-}\Gamma, \Gamma)$ -space (but S is an $S_1(\Gamma, \Gamma)$ -space).
- ▶ If $\mathfrak{b} < \mathfrak{b}_{\mathcal{U}}$ then there is an $S_1(\Gamma, \mathcal{U}\text{-}\Gamma)$ -space which is not an $S_1(\Gamma, \Gamma)$ -space.



\mathcal{I} -quasi-normal convergence

$$f_n \xrightarrow{\mathcal{I}^{\text{QN}}} f$$

there exists $\{\varepsilon_n\}_{n=0}^{\infty}$ \mathcal{I} -converging to 0 such that

$$(\forall x \in X)(\exists A \in \mathcal{I})(\forall n \in \omega)(n \notin A \rightarrow |f_n(x) - f(x)| < \varepsilon_n)$$

$(\mathcal{I}, \mathcal{J})$ wQN-space

X is an $(\mathcal{I}, \mathcal{J})$ wQN-space, if for any sequence $\langle f_n : n \in \omega \rangle$ of continuous functions on X converging to zero there exists a sequence $\langle m_n : n \in \omega \rangle$ such that $f_{m_n} \xrightarrow{\mathcal{J}^{\text{QN}}} 0$.

$$\text{non}((\mathcal{I}, \text{Fin})\text{wQN}) = \min\{\text{cov}^*(\mathcal{I}), \mathfrak{b}\}$$



- ▶ $\text{non}((\text{Fin}, \mathcal{J})\text{wQN}) = \kappa(\mathcal{J})$ for any weak P-ideal \mathcal{J} (i.e., $\text{Fin} \times \text{Fin} \not\leq_K \mathcal{J}$)
- ▶ $\mathfrak{b} \leq \kappa(\mathcal{J}) \leq \mathfrak{d}$ for any weak P-ideal \mathcal{J}
- ▶ $\kappa(\mathcal{J}) = \mathfrak{b}$ for any analytic P-ideal \mathcal{J}



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Thanks for Your attention!