Compactifiability and Borel complexity up to equivalence

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Abstract

We say that two classes $\mathcal{C}$ and $\mathcal{D}$ of topological spaces are *equivalent* if every space in $\mathcal{C}$ is homeomorphic to a space in $\mathcal{D}$ and vice versa. We study the question whether a given family of metrizable compacta (up to the equivalence) can be disjointly composed into one metrizable compact space such that the corresponding quotient space is also a metrizable compactum. We call such families *compactifiable*. For a family of continua this is equivalent to the existence of a metrizable compactum whose set of connected components is equivalent to the original family.

This question is related to the Borel complexity of subsets of the hyperspace of all metrizable compacta $\mathcal{K}([0, 1]^\omega)$. But rather than the complexity of a particular subset, we are interested in the lowest possible complexity among all equivalent subsets.

Every hereditary class of metrizable compacta with a universal element is compactifiable, so compactifiability of a hereditary class might be viewed as a weaker form of existence of a universal element.