On locally compact semitopological graph inverse semigroups

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For a given directed graph $E = (E^0, E^1, r, s)$ a graph inverse semigroup (or simply GIS) $G(E)$ over a graph E is a semigroup with zero generated by the sets E^0, E^1 together with a set $E^{-1} = \{e^{-1} : e \in E^1\}$ satisfying the following relations for all $a, b \in E^0$ and $e, f \in E^1$:

(i) $a \cdot b = a$ if $a = b$ and $a \cdot b = 0$ if $a \neq b$;
(ii) $s(e) \cdot e = e \cdot r(e) = e$;
(iii) $e^{-1} \cdot s(e) = r(e) \cdot e^{-1} = e^{-1}$;
(iv) $e^{-1} \cdot f = r(e)$ if $e = f$ and $e^{-1} \cdot f = 0$ if $e \neq f$.

Graph inverse semigroups are a generalization of polycyclic monoids. In particular, for every non-zero cardinal λ, λ-polycyclic monoid is isomorphic to the graph inverse semigroup over the graph E which consists of one vertex and λ distinct loops.

However, we prove that λ-polycyclic monoid is a universal object in the class of graph inverse semigroups, i.e., each GIS $G(E)$ can be embedded as an inverse subsemigroup into polycyclic monoid $P_{|G(E)|}$. We investigate locally compact (semi)topological graph inverse semigroups. In particular, we characterise those GIS which admit only discrete locally compact semigroup topology. This characterization provides a complete answer on the question of Z. Mesyan, J. D. Mitchell, M. Morayne and Y. H. Péresse. Also we prove that a locally compact semitopological GIS $G(E)$ over a path-complete graph E which contains a finite amount of vertices is either discrete or compact. This result generalizes results of Gutik and Bardyla who proved the above dichotomy for the polycyclic monoids P_1 and P_λ, respectively. Also we investigate embedding of graph inverse semigroup into compact-like (compact, countably compact, feebly compact) topological semigroup and characterises those graph inverse semigroup which can be embedded into compact topological semigroup and countably compact topological inverse semigroup. The above result extended results of Banakh and Gutik who investigates an embedding of the bicyclic monoid into compact-like topological (inverse) semigroup.