

**Serhii Bardyla***Ivan Franko National University of Lviv , Lviv, Ukraine*

For each ordinal  $\alpha$  by the  $\alpha$ -bicyclic monoid  $\mathcal{B}_\alpha$  we mean the set  $\omega^\alpha \times \omega^\alpha$  endowed with the following binary operation:

$$(a, b) \cdot (c, d) = \begin{cases} (a + (c - b), d), & \text{if } b \leq c; \\ (a, d + (b - c)), & \text{if } b > c; \end{cases}$$

We prove that  $\alpha$ -bicyclic monoid  $\mathcal{B}_\alpha$  is algebraically isomorphic to a semigroup of all order isomorphisms between the principal upper sets of the ordinal  $\omega^\alpha$  and prove that  $\mathcal{B}_{\alpha+1}$  is isomorphic to the Brook extension of the semigroup  $\mathcal{B}_\alpha$ . We prove that for every ordinal  $\alpha$  for every  $(a, b) \in \mathcal{B}_\alpha$  if either  $a$  or  $b$  is a non-limit ordinal then  $(a, b)$  is an isolated point in the semitopological  $\mathcal{B}_\alpha$ . We show that for every ordinal  $\alpha < \omega + 1$  every locally compact semigroup topology on  $\mathcal{B}_\alpha$  is discrete. However, we construct an example of a non-discrete locally compact topology  $\tau_{lc}$  on  $\mathcal{B}_{\omega+1}$  such that  $(\mathcal{B}_{\omega+1}, \tau_{lc})$  is a topological inverse semigroup. Also, for every positive integer  $n$  we describe all locally compact topologies on the semitopological  $\mathcal{B}_n$ . In particular we show that there exist exactly  $n$  distinct locally compact topologies on the semitopological  $n$ -bicyclic monoid  $\mathcal{B}_n$ .